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Edexcel

A Level

A Level Maths

Edexcel Core Maths C4 June 2013 Model Solutions

Name:

MME

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Total Marks:

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Edexcel June 2013 C4

1a. $\int x^2 e^x \, dx$ Parts $u = x^2$ $v' = e^x$

$u' = 2x$ $v = e^x$

$= x^2 e^x - \int 2x e^x \, dx \quad (*)$

$\int 2xe^x \, dx$ Parts $u = 2x$ $v' = e^x$

$u' = 2$ $v = e^x$

$= 2xe^x - \int 2e^x \, dx$

$= 2xe^x - 2e^x + c$

Sub into $(*)$

$$\int x^2 e^x \, dx = x^2 e^x - (2xe^x - 2e^x) + c$$

$$= x^2 e^x + 2e^x - 2xe^x + c$$

1b. $$\left[x^2 e^x + 2e^x - 2xe^x \right]_0^1$$

$= (e + 2e - 2e) - (0 + 2 - 0)$

$= e - 2$

2a. $$\sqrt{\frac{(1+x)}{(1-x)}} = (1+x)^{1/2}(1-x)^{-1/2}$$

$$(1+x)^{1/2} \approx 1 + \tfrac{1}{2}x + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}x^2 + \ldots$$

$$= 1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2 + \ldots$$

$$(1-x)^{-1/2} = 1 + \left(-\tfrac{1}{2}\right)(-x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(-x)^2 + \ldots$$

$$= 1 + \tfrac{1}{2}x + \tfrac{3}{8}x^2 + \ldots$$

$$\therefore (1+x)^{1/2}(1-x)^{-1/2} = \left(1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2\right)\left(1 + \tfrac{1}{2}x + \tfrac{3}{8}x^2\right)$$

$$= 1 + \tfrac{1}{2}x + \tfrac{3}{8}x^2 + \tfrac{1}{2}x + \tfrac{1}{4}x^2 - \tfrac{1}{8}x^2 \quad (\text{ignore } x^3 \text{ terms})$$

$$= 1 + x + \tfrac{1}{2}x^2$$

2b. when $x = \frac{1}{26}$

$$\sqrt{\frac{1+x}{1-x}} = \frac{3\sqrt{3}}{5}$$

$$\therefore \frac{3\sqrt{3}}{5} \approx 1 + x + \tfrac{1}{2}x^2 \qquad (\text{when } x = 1/26)$$

$$\sqrt{3} \approx \frac{5}{3}\left(1 + \frac{1}{26} + \frac{1}{2}\cdot\left(\frac{1}{26}\right)^2\right)$$

$$\approx \frac{5}{3} \times \frac{1405}{1352}$$

$$= \frac{7025}{4056}$$

3a. when $x = \pi/3$, $y = 1.154701$

3b. $h = \pi/6$ $\therefore$

$$\int \approx \frac{1}{2}\left(\frac{\pi}{6}\right)\left\{(1 + 1.414214) + 2(1.035276 + 1.154701)\right\}$$

$$= 1.7787 \quad (4\text{ dp})$$

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3c.

$$V = \pi \int_0^{\pi/2} y^2 \, dx \qquad y = \sec(\tfrac{1}{2}x) \qquad y^2 = \sec^2(\tfrac{1}{2}x)$$

$$= \pi \int_0^{\pi/2} \sec^2(\tfrac{1}{2}x) \, dx$$

$$= \pi \left[2\tan(\tfrac{1}{2}x) \right]_0^{\pi/2} \qquad \left(\frac{1}{1/2} = 2 \right)$$

$$= \pi (2 - 0)$$

$$= 2\pi$$

4a.

$$x = 2\sin t \qquad y = 1 - \cos 2t \qquad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$$

$$\frac{dx}{dt} = 2\cos t \qquad \frac{dy}{dt} = 2\sin 2t$$

$$\frac{dy}{dx} = \frac{2\sin 2t}{2\cos t}$$

at $t = \pi/6$, $\dfrac{dy}{dx} = \dfrac{2\sin(\pi/3)}{2\cos(\pi/6)} = 1$

4b

$$y = 1 - \cos 2t$$

$$= 1 - (1 - 2\sin^2 t) \qquad (\cos 2t \equiv 1 - 2\sin^2 t)$$

$$= 2\sin^2 t$$

$$x = 2\sin t$$

$$x^2 = 4\sin^2 t$$

$$\frac{1}{2}x^2 = 2\sin^2 t$$

$$\therefore \quad y = \frac{1}{2}x^2 \qquad (-2 \le x \le 2)$$

4c

$$0 \le f(x) \le 2 \qquad (f(x) \text{ valid for } -2 \le x \le 2)$$

5a. $\int \frac{1}{x(2\sqrt{x}-1)} \, dx$

$x = u^2$

$dx = 2u \, du$

$\sqrt{x} = u$

$= \int \frac{1}{u^2(2u-1)} \cdot 2u \, du$

$= \int \frac{2}{u(2u-1)} \, du$

5b. $\int_1^9 \frac{1}{x(2\sqrt{x}-1)} \, dx$

Limits

x	9	1
u	3	1

$\Rightarrow \int_1^3 \frac{2}{u(2u-1)} \, du$

$\frac{2}{u(2u-1)} = \frac{A}{u} + \frac{B}{2u-1}$

$2 = A(2u-1) + Bu$

$u = 0 \; ; \quad 2 = -A \quad \Rightarrow A = -2$

$u = \frac{1}{2} \; ; \quad 2 = B \cdot \frac{1}{2} \quad \Rightarrow B = 4$

$\int_1^3 -\frac{2}{u} + \frac{4}{2u-1} \, du$

$= \left[-2\ln u + 2\ln|2u-1| \right]_1^3$

$= (-2\ln 3 + 2\ln 5) - (-2\ln 1 + 2\ln 1) \quad (\ln 1 = 0)$

$= 2\ln 5 - 2\ln 3$

$= 2\ln(5/3)$

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6a

$$\frac{d\theta}{dt} = \lambda(120-\theta) \qquad \theta \leq 100$$

$$\int \frac{d\theta}{(120-\theta)} = \int \lambda \, dt$$

$$-\ln|120-\theta| = \lambda t + c$$

when $t=0,\ \theta = 20$

$$-\ln 100 = c$$

$$-\ln|120-\theta| = \lambda t - \ln 100 \qquad (+\ln 100)$$

$$\ln 100 - \ln|120-\theta| = \lambda t$$

$$\ln\left|\frac{100}{120-\theta}\right| = \lambda t$$

$$\frac{100}{120-\theta} = e^{\lambda t}$$

$$120-\theta = \frac{100}{e^{\lambda t}}$$

$$\theta = 120 - \frac{100}{e^{\lambda t}}$$

$$= 120 - 100e^{-\lambda t}$$

6b.

$\lambda = 0.01$, switches off when $\theta = 100$

$$100 = 120 - 100e^{-0.01t}$$

$$100e^{-0.01t} = 20$$

$$e^{-0.01t} = \frac{1}{5}$$

$$-0.01t = \ln \tfrac{1}{5}$$

$$t = \frac{\ln \frac{1}{5}}{-0.01}$$

$$= 161 \quad (3\text{ s.f.})$$

7a. $x^2 + 4xy + y^2 + 27 = 0$ (†)

$2x + 4y + 4x\frac{dy}{dx} + 2y\frac{dy}{dx} + 0 = 0$

$\frac{dy}{dx}(4x + 2y) = -(2x + 4y)$

$\frac{dy}{dx} = \frac{-(2x+4y)}{4x+2y}$

7b // to y axis => grad = ∞

$\frac{dy}{dx} = \infty \Rightarrow 4x + 2y = 0$

$2y = -4x$

$y = -2x$

'Sub $y = -2x$ into (†)'

$x^2 + 4x(-2x) + (-2x)^2 + 27 = 0$

$x^2 - 8x^2 + 4x^2 + 27 = 0$

$3x^2 = 27$

$x^2 = 9$

$x = \pm 3$

, $x < 0 \Rightarrow x = -3$

when $x = -3$, $y = -2(-3)$

$= 6$

$\therefore Q\ (-3, 6)$

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8a. $\underline{r} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ $\quad A \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$ $\quad P \begin{pmatrix} -p \\ 0 \\ 2p \end{pmatrix}$

$\perp \therefore \overrightarrow{PA} \cdot \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} = 0$

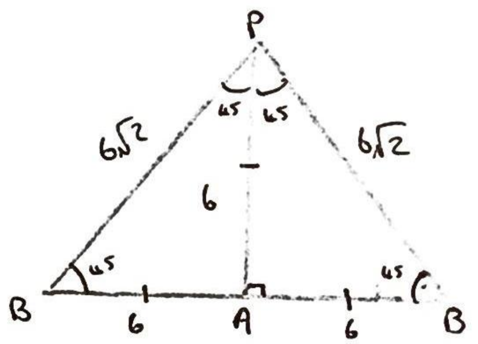
$\overrightarrow{PA} = \begin{pmatrix} 3+p \\ -2 \\ 6-2p \end{pmatrix}$ so $(3+p)2 - 2(2) + (6-2p)(-1) = 0$

$6 + 2p - 4 - 6 + 2p = 0$

$4p - 4 = 0$

$p = 1$

8b.



$|PA| = \sqrt{(3+1)^2 + 2^2 + (6-2)^2} = 6$

$\overrightarrow{OB} = \overrightarrow{OA} \pm 2 \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} \pm \begin{pmatrix} 4 \\ 4 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ 2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -6 \\ 8 \end{pmatrix}$

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