Edexcel

A Level

A Level Maths

Edexcel Core Maths C3 June 2013 Model Solutions

Name:

M M E
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Total Marks:

Edexcel June 13 C3

1.

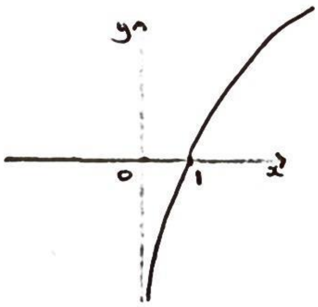
$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} = ax^2 + bx + c + \frac{dx + e}{x^2 - 4}$$

$$\begin{array}{r} 3x^2 - 2x + 7 \\ x^2 + 0x - 4 \overline{) 3x^4 - 2x^3 - 5x^2 + 0x - 4} \\ \underline{3x^4 + 0x^3 - 12x^2} \downarrow \quad \downarrow \\ -2x^3 + 7x^2 + 0x \\ \underline{-2x^3 + 0x^2 + 8x} \\ 7x^2 - 8x - 4 \\ \underline{7x^2 + 0x - 28} \\ -8x + 24 \end{array}$$

$$\therefore \quad 3x^2 - 2x + 7 + \frac{-8x + 24}{x^2 - 4}$$

$a = 3$, $b = -2$, $c = 7$, $d = -8$, $e = 24$

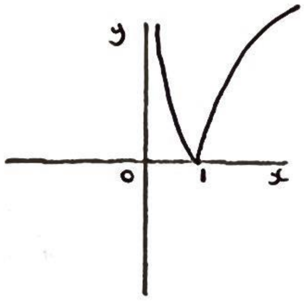
2a.



$y = \ln x$

$x = 0$ asymptote

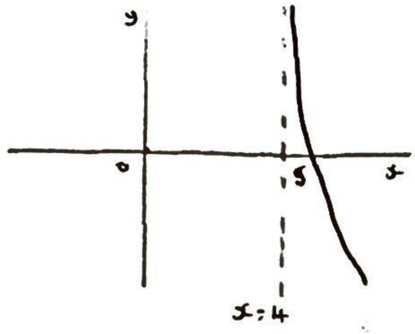
2b.



$y = |\ln x|$

$x = 0$ asymptote

2c.



$y = -f(x-4)$

$x = 4$ asymptote

3a.

$2\cos(x+50) = \sin(x+40)$

$2(\cos x \cos 50 - \sin x \sin 50) = \sin x \cos 40 + \cos x \sin 40$

$2\cos x \cos 50 - 2\sin x \sin 50 = \sin x \cos 40 + \cos x \sin 40$

$\cos 50° \equiv \sin 40°$

$\sin 50° \equiv \cos 40°$

'Replace all 50° with 40° equivalent'

$2\cos x \sin 40 - 2\sin x \cos 40 = \sin x \cos 40 + \cos x \sin 40$

$\div \cos x$

$2\sin 40 - 2\tan x \cos 40 = \tan x \cos 40 + \sin 40$

$2\tan 40 - 2\tan x = \tan x + \tan 40$ $\quad \div \cos 40°$

$3\tan x = \tan 40$

$\tan x = \frac{1}{3}\tan 40$

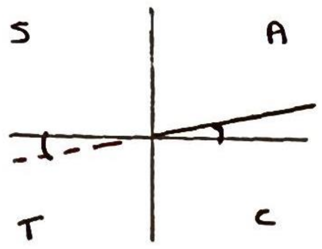
3b.

$\tan 2\theta = \frac{1}{3}\tan 40°$ $\qquad 0 \leq \theta < 360$

let $\phi = 2\theta$ $\qquad 0 \leq \phi < 720$

P.V. $\phi = \tan^{-1}\left(\frac{1}{3}\tan 40\right)$

$= 15.63$



$\phi = 15.63, 195.63, 375.63, 555.63$

$\therefore \quad \theta = 7.8°, 97.8°, 187.8°, 277.8° \text{ (1dp)}$

4a. $f(x) = 25x^2e^{2x} - 16$

$f'(x) = 50xe^{2x} + 50x^2e^{2x}$

at turning points $f'(x) = 0$

$50xe^{2x} + 50x^2e^{2x} = 0 \qquad \div e^{2x} \left(\text{since } e^{2x} > 0 \ \forall x \in \mathbb{R}\right)$

$50x + 50x^2 = 0$

$50x(1 + x) = 0$

$x = 0$ or $x = -1$

when $x = 0$, $f(x) = 25(0)^2e^{2(0)} - 16 = -16$

when $x = -1$, $f(x) = 25(-1)^2e^{2(-1)} - 16 = 25e^{-2} - 16$

$\therefore \quad (0, -16), \quad (-1, 25e^{-2} - 16)$

4b. $25x^2e^{2x} - 16 = 0$

$25x^2e^{2x} = 16$

$25x^2 = \dfrac{16}{e^{2x}} \quad \sqrt{\ } \quad \left(\sqrt{e^{2x}} = e^x \text{ since } e^x \times e^x = e^{2x}\right)$

$5x = \pm \dfrac{4}{e^x}$

$x = \pm \frac{4}{5}e^{-x}$

4c.

$$x_{n+1} = \frac{4}{5}e^{-x_n}$$

$x_0 = 0.5$

$x_1 = 0.485$

$x_2 = 0.492$

$x_3 = 0.489$

4d.

$x = 0.49$ (2dp)

$f(0.485) = -0.487$

$f(0.495) = 0.485$

change of sign $\Rightarrow$ $\alpha = 0.49$ (2dp)

5a.

$x = \sec^2 3y \qquad 0 < y < \pi/6$

$x = (\sec 3y)^2$

$$\frac{dx}{dy} = 2.3\sec 3y \tan 3y\,(\sec 3y)^1$$

$$= 6\tan 3y \sec^2 3y$$

5b.

$$\frac{dy}{dx} = \frac{1}{6\tan 3y \sec^2 3y} \qquad (x = \sec^2 3y)$$

$$= \frac{1}{6x\tan 3y}$$

$1 + \tan^2 x \equiv \sec^2 x$

$\tan^2 3y \equiv \sec^2 3y - 1$

$$= \frac{1}{6x(x^2-1)^{1/2}}$$

$\tan^2 3y \equiv x^2 - 1$

$\tan 3y \equiv (x^2-1)^{1/2}$

5c. $\frac{dy}{dx} = \frac{1}{6x(x-1)^{1/2}}$ $f = 1$ $f' = 0$ $g = 6x(x-1)^{1/2}$

$g' = 6(x-1)^{1/2} + 3x(x-1)^{-1/2}$

$$\frac{d^2y}{dx^2} = \frac{0(6x(x-1)^{1/2}) - 1(6(x-1)^{1/2} + 3x(x-1)^{-1/2})}{(6x(x-1)^{1/2})^2}$$

$$= \frac{-6(x-1)^{1/2} + 3x(x-1)^{-1/2}}{36x^2(x-1)} \quad \times (x-1)^{1/2}$$

$$= \frac{-6(x-1) + 3x}{36x^2(x-1)^{3/2}}$$

$$= \frac{6 - 9x}{36x^2(x-1)^{3/2}} \quad \div 3$$

$$= \frac{2 - 3x}{12x^2(x-1)^{3/2}}$$

6a. $\ln(4-2x) + \ln(9-3x) = 2\ln(x+1)$

$\ln((4-2x)(9-3x)) = \ln((x+1)^2)$

$(4-2x)(9-3x) = (x+1)^2$

$36 - 30x + 6x^2 = x^2 + 2x + 1$

$5x^2 - 32x + 35 = 0$

$(5x - 7)(x - 5) = 0$

$x = 5$ or $7/5$

6b.

$$2^x e^{3x+1} = 10$$

$$\ln\left(2^x e^{3x+1}\right) = \ln 10$$

$$\ln 2^x + \ln\left(e^{3x+1}\right) = \ln 10$$

$$x \ln 2 + 3x + 1 = \ln 10$$

$$x(3 + \ln 2) = -1 + \ln 10$$

$$x = \frac{-1 + \ln 10}{3 + \ln 2}$$

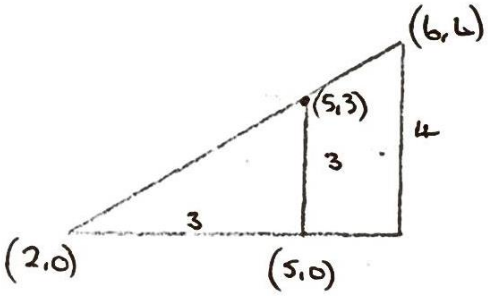
7a.

$$0 \leq f(x) \leq 10$$

7b.

$$ff(0) \quad ; \quad f(0) = 5$$

$$f(5) = 3$$



7c.

$$g(x) = \frac{4 + 3x}{5 - x}$$

let $y = \frac{4+3x}{5-x}$

$$5y - yx = 4 + 3x$$

$$5y - 4 = yx + 3x$$

$$5y - 4 = x(y + 3)$$

$$x = \frac{5y - 4}{y + 3}$$

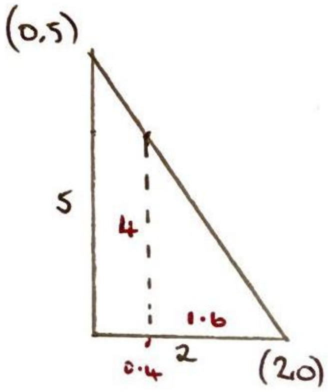
$$\therefore \quad g^{-1}(x) = \frac{5x - 4}{x + 3}$$

7d. $gf(x) = 16$

$g^{-1}\left(gf(x)\right) = g^{-1}(16)$

$f(x) = \dfrac{5(16) - 4}{16 + 3}$

$f(x) = 4 \quad \Rightarrow \quad x = 6 \quad \text{or} \quad x = 0.4$



8a. $24\sin\theta + 7\cos\theta = R\cos(\theta - \alpha)$

$= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$\sin\theta \ ; \quad 24 = R\sin\alpha \quad ①$

$\cos\theta \ ; \quad 7 = R\cos\alpha \quad ②$

'① ÷ ②' $\quad \tan\alpha = \dfrac{24}{7}$

$\alpha = 73.74°$

'sub in ①' $\quad R = \dfrac{24}{\sin(73.74..)} = 25$

$\therefore \quad 24\sin\theta + 7\cos\theta = 25\cos(\theta - 73.74°)$

8b.

$$V = \frac{21}{24\sin\theta + 7\cos\theta}$$

$$= \frac{21}{25\cos(\theta - 73.74)}$$

max value of $\cos(\theta - 73.74) = 1$.

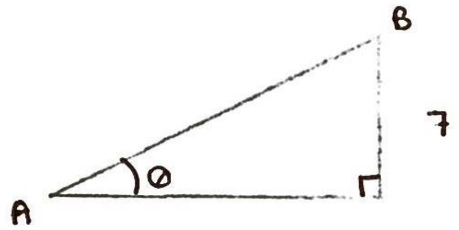
$\therefore$ min. of $V = \frac{21}{25 \times 1} = 0.84$

8c.

when $\cos(\theta - 73.74) = 1$ $(\cos^{-1})$

$\theta - 73.74 = 0$

$\theta = 73.74$



$\sin\theta = \frac{7}{AB}$

$\therefore AB = \frac{7}{\sin(73.74)}$

$= 7.29$ (3sf)

8d.

$$\frac{21}{25\cos(\theta - 73.74)} = 1.68$$

$$\cos(\theta - 73.74) = \frac{21}{25 \times 1.68}$$

$$\cos(\theta - 73.74) = 0.5 \qquad 0 < \theta < 150$$

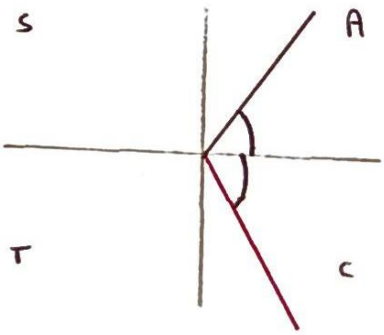
Let $\phi = \theta - 73.74$

$0 < \theta < 150$

$-73.74 < \phi < 76.26$

$\cos\phi = 1/2$

P.V. $\phi = 60°$



$\phi = 60°, \ -60°$

$\theta = 13.7°, \ 133.7° \quad (1\text{ dp})$