Edexcel

A Level

A Level Maths

Edexcel Core Maths C2 June 2013 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

Edexcel June 13 C2

1a. $u_1 = 18 = a$

$u_2 = 12 = ar$

$u_3 = p = ar^2$

$\therefore \quad a = 18. \quad ar = 12$

$18r = 12$

$r = 2/3$

1b. $ar^2 = p$

$18(2/3)^2 = 8$

1c. $S_{15} = \dfrac{a(1-r^{15})}{1-r} \quad ; \quad \dfrac{18(1-(2/3)^{15})}{1-2/3}$

$= 53.877 \quad (3\text{ dp})$

2a. $(2+3x)^4 = {}^4C_0 2^4 \cancel{(3x)} + {}^4C_1 2^3(3x) + {}^4C_2 2^2(3x)^2 + {}^4C_3 2(3x)^3 + (3x)^4$

$= 16 + 96x + 216x^2 + 432x^3 + 81x^4$

2b. $(2-3x)^4 = 16 - 96x + 216x^2 - 432x^3 + 81x^4$

3a. $f(x) = 2x^3 - 5x^2 + ax + 18$

$f(3) = 0 \quad ; \quad 2(3)^3 - 5(3)^2 + a(3) + 18 = 0$

$54 - 45 + 3a + 18 = 0$

$3a = -27$

$a = -9$

3b.

$$\begin{array}{r} 2x^2 + x - 6 \\ x-3 \overline{\big) 2x^3 - 5x^2 - 9x + 18} \\ \underline{2x^3 - 6x^2} \qquad\qquad \\ x^2 - 9x \qquad \\ \underline{x^2 - 3x} \qquad \\ -6x + 18 \\ \underline{-6x + 18} \\ 0 \end{array}$$

$\therefore \quad F(x) = (x-3)(2x^2+x-6)$

$= (x-3)(2x-3)(x+2)$

3c.

$g(y) = 2(3^{3y}) - 5(3^{2y}) - 9(3^{2y}) + 18$

let $x = 3^y$

$x^2 = 3^{2y}$

$x^3 = 3^{3y}$

$g(y) = 2x^3 - 5x^2 - 9x + 18$

$= (x-3)(2x-3)(x+2)$

$x = 3 \quad \therefore \quad 3^y = 3 \quad \Rightarrow \quad y = 1$

$2x = 3 \quad \therefore \quad 3^y = 3/2$

$y \log 3 = \log(3/2) \quad \Rightarrow \quad y = \dfrac{\log(3/2)}{\log 3}$

$= 0.37 \ (2dp)$

$x = -2 \quad \therefore \quad 3^y = -2$ ✗ Impossible

$\therefore \quad y = 1, \ 0.37$

4a.

$x = 1.5 \quad , \quad y = 1.538 \ (3dp)$

4b. $R \approx \frac{1}{2}(0.5)\left\{(5+0.5)+2(4+2.5+1.538+1+0.690)\right\}$

$= 6.239$

4c. $\int_0^3 4 + \frac{5}{(x^2+1)}\,dx = \int_0^3 4\,dx + R$

$= [4x]_0^3 + 6.239$

$= 12. + 6.239$

$= 18.239$

5a. $A = \frac{1}{2}ab\sin C$; $\frac{1}{2}(23)(12)\sin 0.64$

$= 82.4129...$

$= 82.4$ (1dp)

5b. $E\hat{B}C = \pi - 0.64$

$EC = l = r\theta$; $12(\pi - 0.64)$

AE cosine rule : $b^2 = a^2 + e^2 - 2ae\cos B$

$= 23^2 + 12^2 - 2(23)(12)\cos(0.64)$

$b = 15.17376...$

$\therefore P = 23 + 12 + 12(\pi - 0.64) + 15.17376...$

$= 80.1928...$

$= 80.2$ (1dp)

6a. $y = x(x+4)(x-2)$

A $(-4, 0)$ B $(2, 0)$

6b. I_1 (from A - O)

$$\int_{-4}^{0} x(x+4)(x-2) \; dx$$

$$= \int_{-4}^{0} x^3 + 2x^2 - 8x \; dx$$

$$= \left[\tfrac{1}{4}x^4 + \tfrac{2}{3}x^3 - 4x^2\right]_{-4}^{0}$$

$$= 0 - \left(\tfrac{1}{4}(-4)^4 + \tfrac{2}{3}(-4)^3 - 4(-4)^2\right)$$

$$= 0 - (-128/3)$$

$$= 128/3$$

I_2 (from O - B)

$$\int_{0}^{2} x^3 + 2x^2 - 8x \; dx$$

$$= \left[\tfrac{1}{4}x^4 + \tfrac{2}{3}x^3 - 4x^2\right]_{0}^{2}$$

$$= \tfrac{1}{4}(2)^4 + \tfrac{2}{3}(2)^3 - 4(2)^2 - 0$$

$$= -20/3$$

$$\therefore \quad I_2 = 20/3$$

Total shaded area : $\dfrac{128}{3} + \dfrac{20}{3} = \dfrac{148}{3}$

7i.

$$\log_2 2x = \log_2 (5x+4) - 3$$

$$\log_2 \left(\frac{2x}{5x+4}\right) = -3$$

$$\frac{2x}{5x+4} = 2^{-3}$$

$$\frac{2x}{5x+4} = \frac{1}{8}$$

$$16x = 5x + 4$$

$$x = 4/11$$

7ii.

$$\log_a y + 3\log_a 2 = 5$$

$$\log_a y + \log_a 8 = 5$$

$$\log_a 8y = 5$$

$$8y = a^5$$

$$y = \frac{a^5}{8}$$

8i.

$$\tan(x-40) = 1.5$$

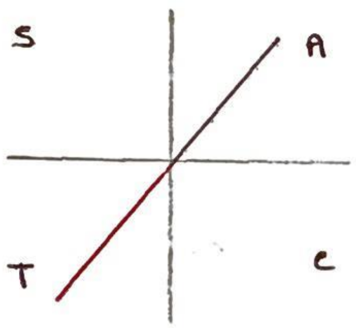
$$-180 \leq x < 180$$

let $\phi = x - 40$

$$-220 \leq \phi < 140$$

$$\tan\phi = 1.5$$

P.V. $\phi = 56.31$



$$\phi = 56.31^\circ, \; -123.69^\circ$$

$$\therefore \; x = 96.3^\circ, \; -83.7^\circ$$

8iia)

$$\sin\theta\tan\theta = 3\cos\theta + 2$$

$$\frac{\sin^2\theta}{\cos\theta} = 3\cos\theta + 2$$

$$\sin^2\theta = \cos\theta(3\cos\theta + 2)$$

$$(1-\cos^2\theta) = 3\cos^2\theta + 2\cos\theta$$

$$4\cos^2\theta + 2\cos\theta - 1 = 0$$

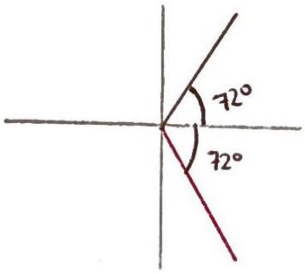
8b. $4\cos^2\theta + 2\cos\theta - 1 = 0 \qquad 0 \leq \theta < 360$

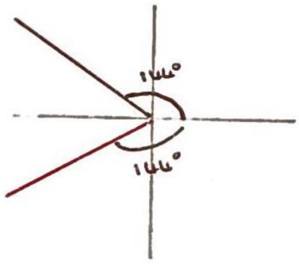
$$\cos\theta = \frac{-2 \pm \sqrt{2^2 - 4(4)(-1)}}{8}$$

$$\cos\theta = \frac{-2 \pm 2\sqrt{5}}{8}$$

$$\cos\theta = \frac{-2 + 2\sqrt{5}}{8} \quad \text{or} \quad \cos\theta = \frac{-2 - 2\sqrt{5}}{8}$$

P.V. $\theta = 72°$ P.V. $\theta = 144°$





$\theta = 72°, 288°$ $\theta = 144°, 216°$

9a. $y = x^2 - 32x^{1/2} + 20$

$$\frac{dy}{dx} = 2x - 16x^{-1/2}$$

at stat. pt. $\frac{dy}{dx} = 0$

$$\frac{16}{\sqrt{x}} = 2x$$

$$16 = 2x^{3/2}$$

$$x^{3/2} = 8$$

$$x = 8^{2/3}$$
$$= 4$$

when $x = 4$, $y = (4)^2 - 32(4)^{1/2} + 20$
$= -28$

$P\ (4, -28)$

9b. $\frac{d^2y}{dx^2} = 2 + 8x^{-3/2}$

$x = 4$, $\frac{d^2y}{dx^2} = 2 + 8(4)^{-3/2}$

$= 3$

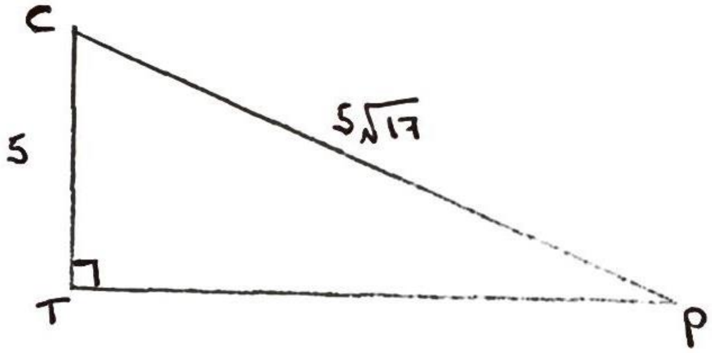
$\frac{d^2y}{dx^2} > 0 \quad \therefore$ minimum

10a. centre is $(-5, 9)$

$C: (x+5)^2 + (y-9)^2 = 5^2$

$|CP| = \sqrt{(8--5)^2 + (-7-9)^2}$

$= 5\sqrt{17}$



$TP^2 = (5\sqrt{17})^2 - 5^2$

$= 400$

$\therefore \quad TP = 20$