Edexcel

A Level

A Level Maths

Edexcel Core Maths C4 June 2012 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

Edexcel June 12 C4

1a.

$$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{3x-1} + \frac{C}{(3x-1)^2}$$

$$1 = A(3x-1)^2 + Bx(3x-1) + Cx$$

$x=0$; $\quad 1 = A$

$x = 1/3$; $\quad 1 = \frac{1}{3}C \Rightarrow C = 3$

$x = 1$; $\quad 1 = 4A + 2B + C$

$$1 = 4 + 2B + 3$$
$$-6 = 2B$$
$$B = -3$$

$$\frac{1}{x(3x-1)^2} = \frac{1}{x} - \frac{3}{3x-1} + \frac{3}{(3x-1)^2}$$

1bi.

$$\int \frac{1}{x} - \frac{3}{3x-1} + \frac{3}{(3x-1)^2}\, dx$$

$$= \ln x - \ln|3x-1| + \int \frac{3}{(3x-1)^2}\, dx$$

$$\frac{d}{dx} k(3x-1)^{-1} = 3(3x-1)^{-2}$$
$$-3k(3x-1)^{-2} = 3(3x-1)^{-2}$$
$$-3k = 3$$
$$k = -1$$

so $$\int \frac{3}{(3x-1)^2}\, dx = -(3x-1)^{-1}$$

so $$\int f(x)\, dx = \ln x - \ln|3x-1| - (3x-1)^{-1} + c$$

1bii. $\left[\ln x - \ln|3x-1| - (3x-1)^{-1}\right]_1^2$

$= \left(\ln 2 - \ln 5 - \frac{1}{5}\right) - \left(\ln 1 - \ln 2 - \frac{1}{2}\right)$

$= 2\ln 2 - \ln 5 + \frac{3}{10}$

$= \ln 2^2 - \ln 5 + \frac{3}{10}$

$= \ln \frac{4}{5} + \frac{3}{10}$

2a. $V = x^3$

$\frac{dV}{dx} = 3x^2$

2b. $\frac{dV}{dt} = 0.048$

$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$

$\frac{dx}{dt} = \frac{1}{3x^2} \times 0.048$

$\frac{dx}{dt} = \frac{0.048}{3x^2}$

when $x = 8$, $\frac{dx}{dt} = \frac{0.048}{3(8)^2} = \frac{1}{4000}$

2c. $S = 6x^2$

$\frac{dS}{dx} = 12x$

$\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt}$

$= 12x \times \frac{0.048}{3x^2}$

when $x = 8$, $\frac{dS}{dt} = 12(8) \times \frac{0.048}{3(8)^2} = \frac{3}{125}$

3a. $f(x) = 6(9-4x)^{-1/2}$

$= 6\left[9(1-4/9x)\right]^{-1/2}$

$= 6 \cdot 9^{-1/2}(1-4/9x)^{-1/2}$

$= 2(1-4/9x)^{-1/2}$

$= 2\left[1 + (-\tfrac{1}{2})(-\tfrac{4}{9}x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-4/9x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{6}(-4/9x)^3 + \dots\right]$

$= 2\left[1 + \frac{2}{9}x + \frac{2}{27}x^2 + \frac{20}{729}x^3 + \dots\right]$

$= 2 + \frac{4}{9}x + \frac{4}{27}x^2 + \frac{40}{729}x^3 + \dots$

3b. $g(x) = 6(9+4x)^{-1/2}$

$= 2 - \frac{4}{9}x + \frac{4}{27}x^2 - \frac{40}{729}x^3 + \dots$

3c. $h(x) = 6(9-8x)^{-1/2}$

'replace x with $2x$'

$= 2 + \frac{4}{9}(2x) + \frac{4}{27}(2x)^2 + \frac{40}{729}(2x)^3 + \dots$

$= 2 + \frac{8}{9}x + \frac{16}{27}x^2 + \frac{320}{729}x^3 + \dots$

4.

$$\frac{dy}{dx} = \frac{3}{y\cos^2 x}$$

$$\int y \, dy = \int 3\sec^2 x \, dx$$

$$\frac{1}{2}y^2 = 3\tan x + c$$

when $x = \frac{\pi}{4}$, $y = 2$

$$\frac{1}{2}(2)^2 = 3\tan\left(\frac{\pi}{4}\right) + c$$

$$2 = 3 + c \implies c = -1$$

$$\frac{1}{2}y^2 = 3\tan x - 1$$

5a.

$$16y^3 + 9x^2y - 54x = 0$$

$$48y^2\frac{dy}{dx} + 18xy + 9x^2\frac{dy}{dx} - 54 = 0$$

$$\frac{dy}{dx}\left(48y^2 + 9x^2\right) = 54 - 18xy$$

$$\frac{dy}{dx} = \frac{54 - 18xy}{48y^2 + 9x^2}$$

5b.

$\frac{dy}{dx} = 0$; $54 - 18xy = 0$

$$18xy = 54$$

$$xy = 3$$

$$x = 3/y$$

Sub $x = \frac{3}{y}$ into C'

$$16y^3 + 9y\left(\frac{3}{y}\right)^2 - 54\left(\frac{3}{y}\right) = 0$$

$$16y^3 + \frac{81y}{y^2} - \frac{162}{y} = 0$$

$$16y^3 + \frac{81}{y} - \frac{162}{y} = 0$$

$$16y^3 - \frac{81}{y} = 0$$

$$16y^4 = 81$$

$$y^4 = \frac{81}{16}$$

$$y = \sqrt[4]{\frac{81}{16}} = \pm\frac{3}{2}$$

$x = 3/y$, when $y = \frac{3}{2}$ $x = 3/\tfrac{3}{2} = 2$

$y = -3/2$ $x = 3/-\tfrac{3}{2} = -2$

so $(2, 3/2)$, $(-2, -3/2)$

6a. $x = \sqrt{3}\sin 2t$, $y = 4\cos^2 t$ $\quad 0 \leq t \leq \pi$

$$\frac{dx}{dt} = 2\sqrt{3}\cos 2t$$

$$y = (2\cos t)^2$$

$$\frac{dy}{dt} = 2.(-2\sin t)(2\cos t)'$$

$$= -8\sin t\cos t$$

$$= -4\sin 2t \quad (\sin 2t \equiv 2\sin t\cos t)$$

$$\frac{dy}{dx} = \frac{-4\sin 2t}{2\sqrt{3}\cos 2t}$$

$$= -\frac{2}{\sqrt{3}}\tan 2t = -\frac{2\sqrt{3}}{3}\tan 2t$$

6b. when $t = \pi/3$

$x = \sqrt{3}\sin\left(\frac{2\pi}{3}\right)$ $\qquad$ $y = 4\cos^2\left(\pi/3\right)$

$= \frac{3}{2}$ $\qquad$ $= 1$

$(3/2, 1)$

when $t = \pi/3$, $\frac{dy}{dx} = -\frac{2\sqrt{3}}{3}\tan\left(\frac{2\pi}{3}\right)$

$= 2$

$y - 1 = 2(x - 3/2)$

$y - 1 = 2x - 3$

$y = 2x - 2$

6c.

$x = \sqrt{3}\sin 2t$

$x^2 = 3\sin^2 2t$ $\qquad$ $y = 4\cos^2 t$

$x^2 = 3(2\cos t\sin t)^2$ $\qquad$ $(\sin 2t \equiv 2\cos t\sin t)$

$x^2 = 12\cos^2 t\sin^2 t$

$\sin^2 t = \frac{x^2}{12\cos^2 t}$ $\qquad$ $y = 4\cos^2 t$

$\cos^2 t = \frac{y}{4}$

$= \frac{x^2}{3y}$

$\sin^2 t + \cos^2 t = 1$

$\frac{x^2}{3y} + \frac{y}{4} = 1$

7a. $\int_1^4 x^{1/2} \ln 2x \; dx \qquad h = \frac{4-1}{3} = 1$

x	y
1	$\ln 2$
2	$\sqrt{2} \ln 4$
3	$\sqrt{3} \ln 6$
4	$2 \ln 8$

$$\int \approx \frac{1}{2}(1)\left\{ (\ln 2 + 2\ln 8) + 2\left(\sqrt{2}\ln 4 + \sqrt{3}\ln 6\right) \right\}$$

$$= 7.49 \quad (2dp)$$

7b. $\int x^{1/2} \ln 2x \; dx$

Parts $\quad u = \ln 2x \qquad v' = x^{1/2}$

$u' = \frac{2}{2x} = \frac{1}{x} \qquad v = \frac{2}{3}x^{3/2}$

$$= \frac{2}{3}x^{3/2}\ln 2x - \int \frac{2}{3}x^{3/2} \cdot \frac{1}{x} \; dx$$

$$= \frac{2}{3}x^{3/2}\ln 2x - \frac{2}{3}\int x^{1/2} \; dx$$

$$= \frac{2}{3}x^{3/2}\ln 2x - \frac{4}{9}x^{3/2} + c$$

7c. $\left[\frac{2}{3}x^{3/2}\ln 2x - \frac{4}{9}x^{3/2} \right]_1^4$

$$= \left(\frac{2}{3}(4)^{3/2}\ln 8 - \frac{4}{9}(4)^{3/2} \right) - \left(\frac{2}{3}\ln 2 - \frac{4}{9} \right)$$

$$= \frac{16}{3}\ln 8 - \frac{32}{9} - \frac{2}{3}\ln 2 + \frac{4}{9}$$

$$= \frac{16}{3}\ln 2^3 - \frac{2}{3}\ln 2 - \frac{28}{9}$$

$$= 16\ln 2 - \frac{2}{3}\ln 2 - \frac{28}{9}$$

$$= \frac{46}{3}\ln 2 - \frac{28}{9}$$

8a. $A\begin{pmatrix}10\\2\\3\end{pmatrix}$ $B\begin{pmatrix}8\\3\\4\end{pmatrix}$

$\vec{AB} = \begin{pmatrix}-2\\1\\1\end{pmatrix}$

8b. l: $\underline{r} = \begin{pmatrix}10\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}-2\\1\\1\end{pmatrix}$

8c. $C\begin{pmatrix}3\\12\\3\end{pmatrix}$

P lies on l so P at $\begin{pmatrix}10-2\lambda\\2+\lambda\\3+\lambda\end{pmatrix}$ for some λ

$\vec{CP} = \begin{pmatrix}10-2\lambda\\2+\lambda\\3+\lambda\end{pmatrix} - \begin{pmatrix}3\\12\\3\end{pmatrix}$

$= \begin{pmatrix}7-2\lambda\\-10+\lambda\\\lambda\end{pmatrix}$

$\perp$ so $\vec{CP} \cdot \vec{AB} = 0$

$\begin{pmatrix}7-2\lambda\\-10+\lambda\\\lambda\end{pmatrix} \cdot \begin{pmatrix}-2\\1\\1\end{pmatrix} = 0$

$-2(7-2\lambda) + (-10+\lambda) + \lambda = 0$

$-14 + 4\lambda - 10 + \lambda + \lambda = 0$

$6\lambda = 24$

$\lambda = 4$

so P at $\begin{pmatrix}10-2(4)\\2+4\\3+4\end{pmatrix}$

$= (2, 6, 7)$