Edexcel

A Level

A Level Maths

Edexcel Core Maths C3 June 2012 Model Solutions

Name:

M M E

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Total Marks:

Edexcel June 12 C3

1. $\dfrac{2(3x+2)}{9x^2-4} - \dfrac{2}{3x+1}$

$\dfrac{2(3x+2)}{(3x+2)(3x-2)} - \dfrac{2}{3x+1}$

$\dfrac{2(3x+1) - 2(3x-2)}{(3x+1)(3x-2)}$, $\dfrac{6}{(3x+1)(3x+2)}$

2a. $f(x) = x^3 + 3x^2 + 4x - 12$

$x^3 + 3x^2 = 12 - 4x$
$x^2(x+3) = 4(3-x)$

$x = \sqrt{\dfrac{4(3-x)}{x+3}}$, $x \neq -3$

2b. $x_0 = 1$
$x_1 = 1.41$
$x_2 = 1.20$
$x_3 = 1.31$

2c. $\alpha = 1.272$ to 3dp. $\Rightarrow \alpha \in [1.2715, 1.2725)$

$f(1.2715) = 0.00827$
$f(1.2725) = -0.00821$

change of sign $\Rightarrow \alpha \in (1.2715, 1.2725)$
$\Rightarrow \alpha = 1.272$ to 3dp.

3a. $\quad y = e^{x\sqrt{3}} \sin 3x$, $\qquad -\dfrac{\pi}{3} \le x \le \dfrac{\pi}{3}$

$$-\pi \le 3x \le \pi$$

$$\dfrac{dy}{dx} = \sqrt{3}\, e^{x\sqrt{3}} \sin 3x + 3 e^{x\sqrt{3}} \cos 3x$$

for turning point, $\dfrac{dy}{dx} = 0$

$$e^{x\sqrt{3}}\left(\sqrt{3}\sin 3x + 3\cos 3x\right) = 0 \qquad \left(e^{x\sqrt{3}} > 0\right)$$

$$\sqrt{3}\sin 3x = -3\cos 3x$$

$$\tan 3x = -\dfrac{3}{\sqrt{3}}$$

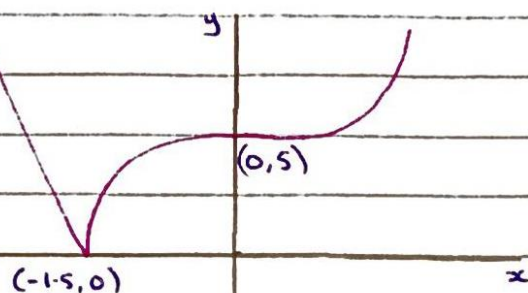P.V. $\quad -\dfrac{1}{3}\pi, \quad \dfrac{2}{3}\pi$
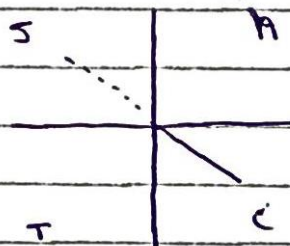
$$x = -\dfrac{1}{9}\pi, \quad \dfrac{2}{9}\pi$$

since $x > 0$, $\quad x = \dfrac{2}{9}\pi$

3b. $\quad x = 0, \quad y = 0$

$$\dfrac{dy}{dx} = \sqrt{3}\, e^{0} \sin(0) + 3 e^{0} \cos(0) = 3$$

$$y = -\dfrac{1}{3}x$$

4a.

$y = |f(x)|$

(0,5)

(-1.5, 0)

4b.

$y = f(|x|)$

(0,5)

4c.   stretch x s.f. $\frac{1}{3}$ $y = 2f(3x)$
       stretch y s.f. 2

(0, 10)

(-0.5, 0)
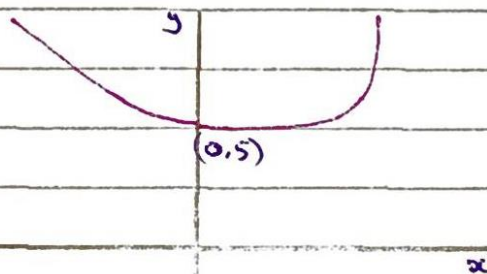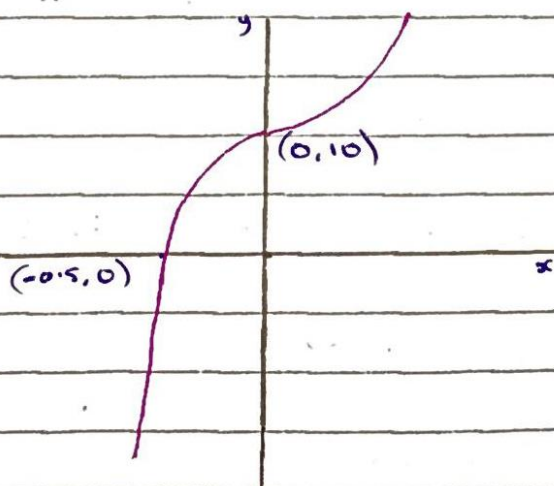
5a. $4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta$. $\dfrac{4}{\sin^2 2\theta}$ $\dfrac{1}{\sin^2\theta}$

use $\sin 2\theta \equiv 2\sin\theta\cos\theta$

$\sin^2 2\theta \equiv 4\sin^2\theta\cos^2\theta$

$\dfrac{4}{4\sin^2\theta\cos^2\theta}$ $\dfrac{1}{\sin^2\theta}$

5b. $4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta \equiv \sec^2\theta$

LHS : $\dfrac{1}{\sin^2\theta\cos^2\theta} - \dfrac{1}{\sin^2\theta}$

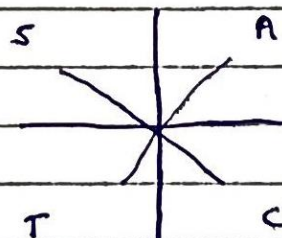$= \dfrac{1 - \cos^2\theta}{\sin^2\theta\cos^2\theta}$ $1 - \cos^2\theta \equiv \sin^2\theta$

$= \dfrac{\sin^2\theta}{\sin^2\theta\cos^2\theta} \cdot \dfrac{1}{\cos^2\theta} = \sec^2\theta = $ RHS

5c. $4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta = 4$ $0 < \theta < \pi$

$\sec^2\theta = 4$

$\cos\theta = \pm \frac{1}{2}$

$\theta = \dfrac{\pi}{3} , \dfrac{2\pi}{3}$

6a.　　　$f(x) = e^x + 2$　　　　$x \in \mathbb{R}$

　　　　$g(x) = \ln x$　　　　$x > 0$

　　$f(x) > 2$　　(since $e^x > 0$)

6b.　　$fg(x) = f(\ln x) = e^{\ln x} + 2 = x + 2$

6c.　　$f(2x + 3) = 6$

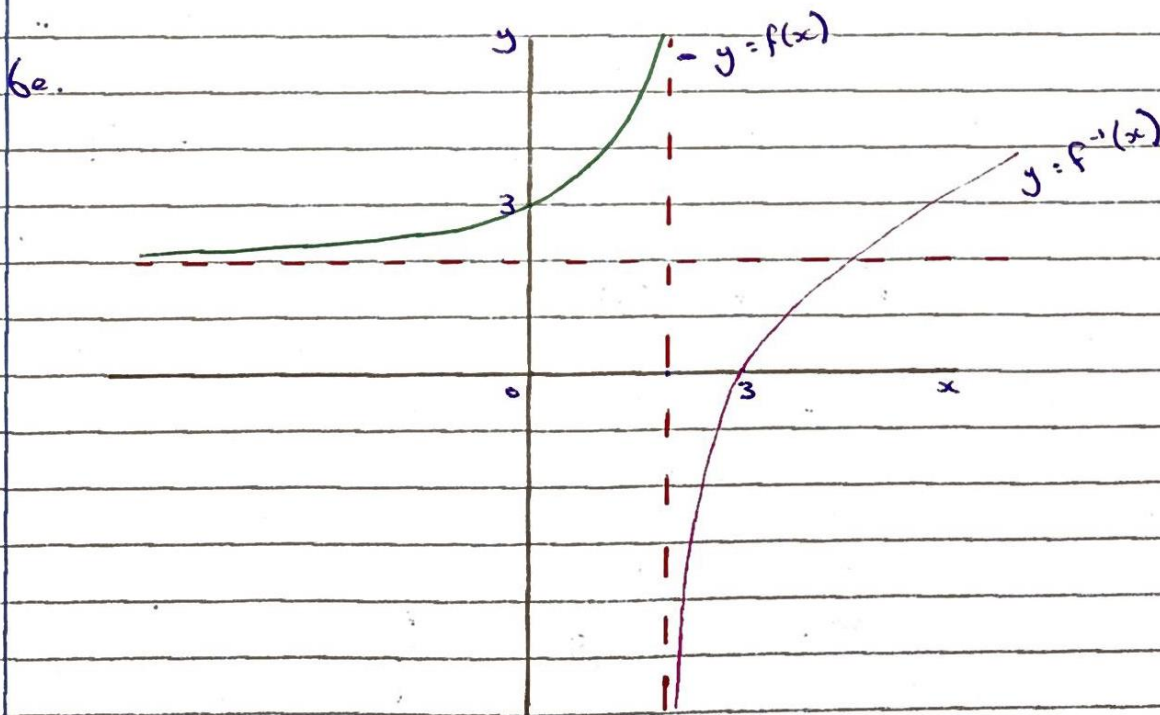　　　$e^{2x+3} + 2 = 6$

　　　$2x + 3 = \ln 4$

　　　　$x = \frac{1}{2}(\ln 4 - 3)$

6d.　　　let　$y = e^{x+2} + 2$

　　　　　$x = \ln(y-2)$

　　　$f^{-1}(x) = \ln(x-2)$

6e.

7a. $\quad \dfrac{d}{dx} \; x^{1/2} \ln 3x \; = \; \dfrac{1}{2} x^{-1/2} \cdot \ln 3x \; + \; \dfrac{3x^{1/2}}{3x}$

7b. $\quad \dfrac{d}{dx}\left[\dfrac{1-10x}{(2x-1)^5}\right] \qquad\qquad f = 1-10x \qquad g = (2x-1)^5$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad f' = -10 \qquad g' = 10(2x-1)^4$

$\qquad = \quad \dfrac{-10(2x-1)^5 - 10(1-10x)(2x-1)^4}{(2x-1)^{10}}$

$\qquad = \quad \dfrac{-10(2x-1) - 10(1-10x)}{(2x-1)^6}$

$\qquad = \quad \dfrac{-20x + 10 - 10 + 100x}{(2x-1)^6}$

$\qquad = \quad \dfrac{80x}{(2x-1)^6}$

7c. $\quad x = 3\tan(2y) \qquad\qquad\qquad \left(\dfrac{x}{3}\right)^2 = \tan^2(2y)$

$\qquad 1 = 6\sec^2(2y)\,\dfrac{dy}{dx} \qquad\qquad \sec^2(2y) = 1 + \tan^2(2y)$

$\qquad \dfrac{dy}{dx} = \dfrac{1}{6\sec^2(2y)} \qquad\qquad\qquad = 1 + \left(\dfrac{x}{3}\right)^2$

$\qquad = \dfrac{1}{6\left(1+\left(\dfrac{x}{3}\right)^2\right)}$

$\qquad = \dfrac{1}{6 + 6x^2/9}$

$\qquad = \dfrac{9}{54 + 6x^2} \qquad = \dfrac{3}{18 + 2x^2}$

8a. $\quad F(x) = 7\cos 2x - 24\sin 2x$

$$7\cos 2x - 24\sin 2x = R\cos(2x + \alpha)$$
$$= R(\cos 2x \cos\alpha - \sin 2x \sin\alpha)$$

$$R = \sqrt{7^2 + (-24)^2} = 25$$

$\cos 2x$; $\quad 7 = 25\cos\alpha \quad \Rightarrow \quad \alpha = 73.74°$

8b. $\quad 25\cos(2x + 73.74°) = 12.5 \qquad 0 \le x \le 180°$

$$2x + 73.74° = \cos^{-1}\left(\frac{12.5}{25}\right)$$

$$73.74 \le 2x + 73.74$$
$$\le 433.74$$

P.V. $\quad 60°, \, 300°, \, 420°$

$$2x + 73.74 = \begin{cases} 60° & \Rightarrow \quad x = -6.87° \\ 300° & \Rightarrow \quad x = 113.13° \\ 420° & \Rightarrow \quad x = 173.145° \end{cases}$$

$$\Rightarrow \quad x = 113.1° , \, 173.1°$$

8c. $14\cos^2 x - 48\sin x \cos x$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = 2\cos^2 x - 1$

~~$28\cos^2 x$~~

$7\cos 2x + 7 - 24\sin 2x$

8d. $14\cos^2 x - 48\sin x \cos x ~~+7~~ = f(x) + 7$

$25 \cos(2x + 73.74°) + 7$

$\Rightarrow$ max value is 32