Edexcel

A Level

A Level Maths

Edexcel Core Maths C2 June 2012 Model Solutions

Name:

Total Marks:

Edexcel June 12 C2

1. $(2-3x)^5 = {}^5C_0\, 2^5 + {}^5C_1\, 2^4(-3x) + {}^5C_2\, 2^3(-3x)^2 + \ldots$

$= 32 - 240x + 720x^2 + \ldots$

2a. $2\log_3 x - \log_3(x-2) = 2$

$\log_3 x^2 - \log_3(x-2) = 2$

$\log_3\left(\frac{x^2}{x-2}\right) = 2$

$\frac{x^2}{x-2} = 3^2$

$x^2 = 9(x-2)$

$x^2 - 9x + 18 = 0$

$(x-6)(x-3) = 0$

$x = 6$ or 3

3a. $x^2 + y^2 - 20x - 16y + 139 = 0$

$(x-10)^2 - 100 + (y-8)^2 - 64 + 139 = 0$

$(x-10)^2 + (y-8)^2 = 25 \quad (= r^2)$

C at $(10, 8)$

3b. $r = \sqrt{25}$

$= 5$

3c. $x = 13$; $(13-10)^2 + (y-8)^2 = 25$

$9 + (y-8)^2 = 25$

$y - 8 = \sqrt{16}$

$y = 8 \pm 4$

$= 12$ or 4

Q $(13, 4)$ P $(13, 12)$

3d. $l = r\theta$; $5(1.855) = 9.275$

$P = 2r + 9.275$

$= 19.3$ (3 s.f.)

4a. $f(x) = 2x^3 - 7x^2 - 10x + 24$

$f(-2) = 2(-2)^3 - 7(-2)^2 - 10(-2) + 24$

$= -16 - 28 + 20 + 24$

$= -44 + 44$

$= 0$ $\therefore$ $(x+2)$ is a factor

4b.

$$\begin{array}{r|l} & 2x^2 - 11x + 12 \\ \hline x+2 & 2x^3 - 7x^2 - 10x + 24 \\ & \underline{2x^3 + 4x^2} \\ & -11x^2 - 10x \\ & \underline{-11x^2 - 22x} \\ & 12x + 24 \\ & \underline{12x + 24} \\ & 0 \end{array}$$

$\therefore$ $f(x) = (x+2)(2x^2 - 11x + 12)$

$= (x+2)(2x-3)(x-4)$

5a. $y = 10 - x$, $y = 10x - x^2 - 8$

$10 - x = 10x - x^2 - 8$

$x^2 - 11x + 18 = 0$

$(x-2)(x-9) = 0$

$x = 2$ or $x = 9$

$x = 2$, $y = 10 - 2 = 8$ A $(2, 8)$

$x = 9$, $y = 10 - 9 = 1$ B $(9, 1)$

5b.

$$R = \int_2^9 (10x - x^2 - 8) - (10 - x) \ dx$$

$$= \int_2^9 -x^2 + 11x - 18 \ dx$$

$$= \left[-\tfrac{1}{3}x^3 + \tfrac{11}{2}x^2 - 18x \right]_2^9$$

$$= \left(-\tfrac{1}{3}(9)^3 + \tfrac{11}{2}(9)^2 - 18(9)\right) - \left(-\tfrac{1}{3}(2)^3 + \tfrac{11}{2}(2)^2 - 18(2)\right)$$

$$= \frac{81}{2} - \left(-\frac{50}{3}\right)$$

$$= \frac{343}{6}$$

6a.

$$\tan 2x = 5 \sin 2x$$

$$\frac{\sin 2x}{\cos 2x} = 5 \sin 2x$$

$$\sin 2x = 5 \sin 2x \cos 2x$$

$$0 = 5 \sin 2x \cos 2x - \sin 2x$$

$$(1 - 5\cos 2x)\sin 2x = 0$$

6b.

$$\sin 2x (1 - 5\cos 2x) = 0$$

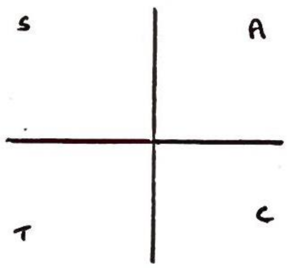
$0 \leq x \leq 180°$

$\sin 2x = 0 \qquad 0 \leq x \leq 180$

Let $\phi = 2x \qquad 0 \leq \phi \leq 360$

$\sin \phi = 0$

P.V. $\phi = 0$



$\phi = 0, 180°, 360°$

$x = 0°, 90°, 180°$

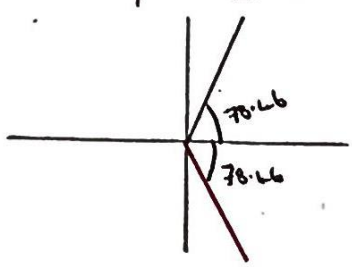
or

$5\cos 2x = 1$

$\cos 2x = 1/5 \qquad 0 \leq \phi \leq 360$

$\cos \phi = 1/5$

P.V. $\phi = 78.46$



$\phi = 78.46, \ 281.53°$

$x = 39.2°, \ 140.8°$

7a. $x = 0.5, \quad y = 1.494$

$x = 0.75, \quad y = 1.741$

7b. $h = 0.25$

$$\int \approx \frac{1}{2}(0.25)\left\{(1+2) + 2(1.251 + 1.494 + 1.741)\right\}$$

$$= 1.4965$$

8a. $V = \pi r^2 h$

$60 = \pi x^2 h$

$h = \dfrac{60}{\pi x^2}$

8b. $A = 2\pi r^2 + 2\pi r h$

$= 2\pi x^2 + 2\pi x\left(\dfrac{60}{\pi x^2}\right)$

$= 2\pi x^2 + \dfrac{120}{x}$

8c. $\dfrac{dA}{dx} = 4\pi x - 120x^{-2}$

for minimum, $\dfrac{dA}{dx} = 0$

$\dfrac{120}{x^2} = 4\pi x$

$120 = 4\pi x^3$

$x = \sqrt[3]{\dfrac{120}{4\pi}}$

8d. $A = 2\pi\left(\sqrt[3]{\dfrac{120}{4\pi}}\right)^2 + \dfrac{120}{\sqrt[3]{\dfrac{120}{4\pi}}}$

$= 84.84\ldots$

$= 85$ (nearest integer)

8e.
$$\frac{d^2A}{dx^2} = 4\pi + 240x^{-3}$$

when $x = \sqrt[3]{\frac{120}{4\pi}}$, $\frac{d^2A}{dx^2} = 12\pi$

$12\pi > 0$ ∴ minimum

9a.
$$S_n = a + \cancel{ar} + \cancel{ar^2} + \cancel{\ldots} + \cancel{ar^{n-2}} + \cancel{ar^{n-1}} \quad ①$$

'Multiply ① by r'

$$rS_n = \cancel{ar} + \cancel{ar^2} + \cancel{ar^3} + \ldots + \cancel{ar^{n-1}} + ar^n \quad ②$$

'① - ②'

$$S_n - rS_n = a - ar^n$$
$$S_n(1-r) = a(1-r^n)$$
$$S_n = \frac{a(1-r^n)}{(1-r)}$$

9b.
$u_3 = 5.4 \Rightarrow ar^2 = 5.4$ ①

$u_5 = 1.944 \Rightarrow ar^4 = 1.944$ ②

'② ÷ ①'
$$\frac{ar^4}{ar^2} = \frac{1.944}{5.4}$$
$$r^2 = 9/25$$
$$r = 0.6$$

9c.
$a(0.6)^2 = 5.4$; $a = \frac{5.4}{(0.6)^2} = 15$

9d.
$$S_\infty = \frac{a}{1-r} = \frac{15}{1-0.6} = 37.5$$