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Edexcel

A Level

A Level Maths

Edexcel Core Maths C4 June 2011 Model Solutions

Name:

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Total Marks:

Jun 11 Edexcel - C4

1. $$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{2x+1}$$

$$9x^2 = A(x-1)(2x+1) + B(2x+1) + (x-1)^2 C$$

$x = 1$; $9 = 3B \Rightarrow B = 3$

$x = -1/2$; $9/4 = 9/4\, C \Rightarrow C = 1$

$x = 0$; $0 = -A + B + C$

$A = 3 + 1 \Rightarrow A = 4$

2. $$f(x) = (9 + 4x^2)^{-1/2}$$

$$= 9^{-\frac{1}{2}}\left(1 + 4/9\, x^2\right)^{-1/2}$$

$$= \frac{1}{3}\left[1 + (-1/2)(4/9\, x^2) + \frac{(-1/2)(-3/2)}{2!}(4/9\, x^2)^2 + \ldots\right.$$

$$= \frac{1}{3}\left(1 - \frac{2}{9}x^2 + \frac{2}{27}x^4 + \ldots\right.$$

$$= \frac{1}{3} - \frac{2}{27}x^2 + \frac{2}{81}x^4$$

3a. $V = \frac{1}{12}\pi h^2(3-4h)$ $\quad \frac{dV}{dh} = \frac{2h}{12}\pi(3-4h) - \frac{4}{12}\pi h^2$

when $h = 0.1$, $\frac{dV}{dh} = \frac{\pi}{25}$

3b. $$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV} = \frac{\pi}{800} \times \frac{1}{\frac{2h}{12}\pi(3-4h) - \frac{4}{12}\pi h^2}$$

when $h = 0.1$; $\frac{dh}{dt} = \frac{\pi}{800} \times \frac{25}{\pi} = \frac{1}{32}$

4a. $x = \frac{\sqrt{2}}{4}$; $y = 0.0333$

$x = \frac{3\sqrt{2}}{4}$; $y = 1.3596$

4b. $h = \frac{b-a}{n} = \frac{\sqrt{2}}{4}$

$$\text{Area} \approx \frac{1}{2} \cdot \frac{\sqrt{2}}{4} \left\{ (0 + 3.9210) + 2(0.0333 + 1.3596 + 0.3240) \right\}$$

$= 1.300157\ldots$

$= 1.30$ to 2 d.p.

4c. $\frac{1}{2}\int_2^4 (u-2)\ln u \; du$ $\quad u = x^2 + 2$, $du = 2x \; dx$

u	4	2
x	$\sqrt{2}$	0

$$\frac{1}{2}\int_0^{\sqrt{2}} x^2 . \ln(x^2+2) . 2x \; dx$$

$$= \int_0^{\sqrt{2}} x^3 \ln(x^2+2) \; dx = \text{area of } R$$

4d. $\frac{1}{2}\int_2^4 (u-2)\ln u \; du$ $\quad u = \ln u$, $v' = u - 2$; $u' = \frac{1}{u}$, $v = \frac{1}{2}u^2 - 2u$

$$= \frac{1}{2}\left[\ln u\left(\frac{1}{2}u^2 - 2u\right) - \int \frac{1}{u}\left(\frac{1}{2}u^2 - 2u\right) du \right]$$

$$= \frac{1}{2}\left[\left(\frac{1}{2}u^2 - 2u\right)\ln u - \frac{1}{4}u^2 + 2u \right]_2^4$$

$$= \frac{1}{2}\left(4 + (-2\ln 2 + 3)\right) = \frac{1}{2}\left(1 + 2\ln 2\right)$$

5. $\ln y = 2x\ln x \quad \Rightarrow \quad y = e^{2x\ln x}$

$$\frac{1}{y}\frac{dy}{dx} = 2\ln x + 2$$

$$\frac{dy}{dx} = e^{2x\ln x}(2\ln x + 2)$$

at $x = 2$ $\quad \frac{dy}{dx} = e^{2(2)\ln 2}(2\ln 2 + 2)$

$$= 16(2\ln 2 + 2)$$

$$= 32\ln 2 + 32$$

6a. $\underline{r}_1 = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \qquad \underline{r}_2 = \begin{pmatrix} -5 \\ +15 \\ 3 \end{pmatrix} + m \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$

$6 - \lambda = -5 + 2m$ ① $\Rightarrow$ $11 - \lambda = 2m$ $(\times 2)$

$-3 + 2\lambda = +15 - 3m$ ② $\Rightarrow$ $-18 + 2\lambda = -3m$ $(\times 3)$

$-2 + 3\lambda = 3 + m$ $\qquad 22 - 2\lambda = 4m$

$4 = m$

$\lambda = 3$

so A is at $\begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}$

6b. $\begin{pmatrix}-1\\2\\3\end{pmatrix} \cdot \begin{pmatrix}2\\-3\\1\end{pmatrix} = \cos\theta \left|\begin{matrix}-1\\2\\3\end{matrix}\right| \times \left|\begin{matrix}2\\-3\\1\end{matrix}\right|$

$\dfrac{-2-6+3}{\sqrt{14}\cdot\sqrt{14}} = \cos\theta$

$\theta = 110.9^\circ \Rightarrow$ acute angle $= 69.1^\circ$

6c. $B \begin{pmatrix}5\\-1\\1\end{pmatrix}$

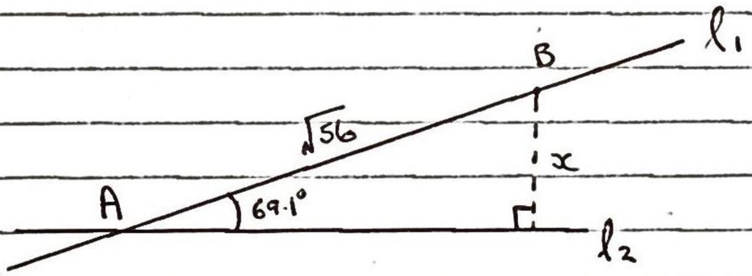
$6-\lambda = 5$

$-3+2\lambda = -1$

$-2+3\lambda = 1$

works for $\lambda = 1 \Rightarrow B$ is on l_1

6d.



$\overrightarrow{AB} = \begin{pmatrix}2\\-4\\-6\end{pmatrix}$ $\quad |AB| = \sqrt{2^2+(-4)^2+(-6)^2} = \sqrt{56}$

T C S
O A O
A H H

$\sin 69.1^\circ = \dfrac{x}{\sqrt{56}}$

$x = \sqrt{56}\cdot\sin 69.1^\circ$

$= 6.99$ to 3 s.f.

8a. $\int (4y+3)^{-1/2} \, dy$

$$\frac{d}{dy}\left[k(4y+3)^{1/2}\right] = \frac{4k}{2}(4y+3)^{-1/2}$$

$$\frac{4k}{2} = 1 \Rightarrow k = 1/2$$

$$\frac{1}{2}(4y+3)^{1/2} + c$$

8b. $$\frac{dy}{dx} = \frac{(4y+3)^{1/2}}{x^2}$$

$$\int \frac{dy}{(4y+3)^{1/2}} = \int \frac{dx}{x^2}$$

$$\frac{1}{2}(4y+3)^{1/2} = -\frac{1}{x} + c$$

$y = 1.5,\ x = -2;$ $\quad \frac{3}{2} = \frac{1}{2} + c \Rightarrow c = 1$

$$\frac{1}{2}(4y+3)^{1/2} = -\frac{1}{x} + 1$$

$$(4y+3)^{1/2} = -\frac{2}{x} + 2$$

$$4y + 3 = \left(-\frac{2}{x} + 2\right)^2$$

$$y = \frac{1}{4}\left(2 - \frac{2}{x}\right)^2 - \frac{3}{4}$$