Edexcel

A Level

A Level Maths

Edexcel Core Maths C3 June 2011 Model Solutions

Name:

MME
Mathsmadeeasy.co.uk

Total Marks:

Edexcel June 11 C3

1a. $\frac{d}{dx}\left[\ln(x^2+3x+5)\right] = \frac{2x+3}{x^2+3x+5}$

since $\frac{d}{dx}\left[\ln(f(x))\right] = \frac{f'(x)}{f(x)}$

1b. $\frac{d}{dx}\left[\frac{\cos x}{x^2}\right]$; $f = \cos x$ $\quad g = x^2$

$f' = -\sin x \quad g' = 2x$

$= \frac{-\sin x \cdot x^2 - \cos x 2x}{(x^2)^2}$

$= \frac{-x^2\sin x - 2x\cos x}{x^4}$

$= \frac{-x\sin x - 2\cos x}{x^3}$

2a. $f(x) = 2\sin(x^2) + x - 2 \quad , \quad 0 \leq x < 2\pi$

$f(0.75) = -0.18339\ldots$

$f(0.85) = 0.1725\ldots$

change of sign $\therefore$ $0.75 < \alpha < 0.85$ (α is root)

2b. $x_{n+1} = \left[\arcsin(1-0.5x_n)\right]^{1/2}$

$x_0 = 0.8$

$x_1 = 0.80219$

$x_2 = 0.80133$

$x_3 = 0.80167$

2c. if $\alpha = 0.80157$ to 5dp $\quad 0.801565 < \alpha < 0.801575$

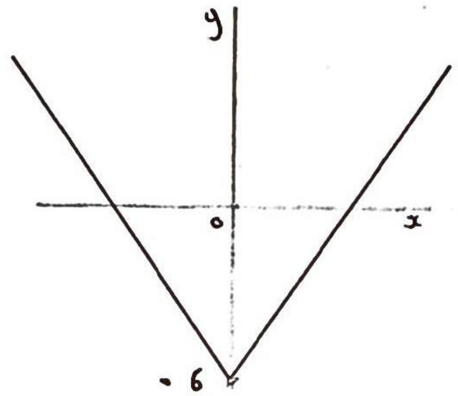
$F(0.801565) = -0.000027...$

$f(0.801575) = 0.00000862...$

change of sign $\therefore 0.801565 < \alpha < 0.801575$

$\therefore \alpha = 0.80157$ to 5 dp.

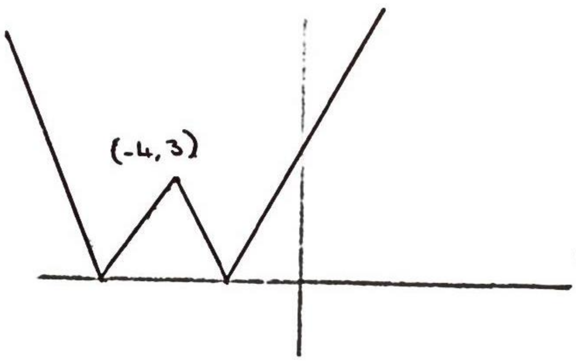
3a.



$y = 2f(x+4)$

R $(0,-6)$

3b.



$y = |f(-x)|$

R $(-4,3)$

4a. $f(x) = 4 - \ln(x+2) \quad , \quad x \in \mathbb{R} \quad , \quad x \geq -1$

let $y = 4 - \ln(x+2)$

$\ln(x+2) = 4 - y$

$x + 2 = e^{4-y}$

$x = e^{4-y} - 2$

$\therefore f^{-1}(x) = e^{4-x} - 2$

4b. domain of $f^{-1}(x)$ = range of $f(x)$

$f(x) = 4 - \ln(x+2) \qquad x \geqslant -1$

$f(-1) = 4 - \ln(1) = 4$

$\therefore \quad f(x) \leq 4$

$\Rightarrow$ domain of $f^{-1}(x)$: $x \leq 4$

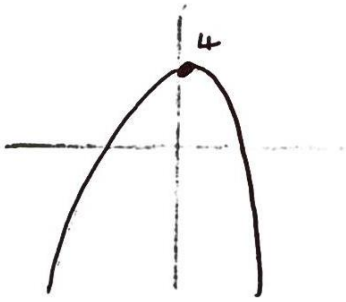
4c. $f(x) = 4 - \ln(x+2)$, $g(x) = e^{x^2} - 2$

$fg(x) = 4 - \ln\left(e^{x^2} - 2 + 2\right)$

$= 4 - \ln\left(e^{x^2}\right)$

$= 4 - \cancel{e^{x^3}}\, x^2$

4d.



range : $fg(x) \leq 4$

5a. when $t = 0$, $m = 7.5$ $\Rightarrow$ $p = 7.5$

5b. $m = 7.5e^{-kt}$

$t = 4$, $m = 2.5$

$2.5 = 7.5e^{-4k}$

$\frac{1}{3} = e^{-4k}$

$\ln \frac{1}{3} = -4k$

$-\ln 3 = -4k$

$k = \frac{1}{4}\ln 3$

5c.

$$m = 7.5\, e^{-\frac{1}{4}\ln 3\, t}$$

$$\frac{dm}{dt} = 7.5\left(-\tfrac{1}{4}\ln 3\right) e^{-\frac{1}{4}\ln 3\, t}$$

$$-0.6\ln 3 = -\frac{15}{8}\ln 3\, e^{-\frac{1}{4}\ln 3\, t}$$

$$\frac{-0.6\ln 3}{-15/8\,\ln 3} = e^{-\frac{1}{4}\ln 3\, t}$$

$$\frac{8}{25} = e^{-\frac{1}{4}\ln 3\quad t}$$

$$\ln\left(\tfrac{8}{25}\right) = -\tfrac{1}{4}\ln 3\quad t$$

$$t = \frac{\ln(8/25)}{-1/4\,\ln 3}$$

$$= 4.15 \quad (3sf)$$

6a.

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan\theta$$

LHS $\quad \dfrac{1-\cos 2\theta}{\sin 2\theta} = \tan\theta$

$$= \frac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta} \qquad \begin{matrix}(\cos 2\theta \equiv 1-2\sin^2\theta) \\ (\sin 2\theta \equiv 2\sin\theta\cos\theta)\end{matrix}$$

$$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta = \text{RHS}$$

6bi:

$$\tan 15^\circ = \frac{1}{\sin 30^\circ} - \frac{\cos 30^\circ}{\sin 30^\circ} \qquad (\theta = 15^\circ)$$

$$= \frac{1}{1/2} - \frac{\sqrt{3}/2}{1/2}$$

$$= 2 - \sqrt{3}$$

6bii.

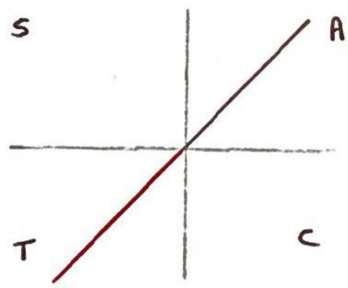
$$\operatorname{cosec} 4x - \cot 4x = 1 \qquad 0 < x < 360^\circ$$

$$\frac{1}{\sin 4x} - \frac{\cos 4x}{\sin 4x} = 1$$

let $\theta = 2x$

$$\tan \theta = 1 \qquad 0 < \theta < 720$$

P.V. $\theta = 45^\circ$



$\theta = 45^\circ, 225^\circ, 405^\circ, 585^\circ$

$x = 22.5^\circ, 112.5^\circ, 202.5^\circ, 292.5^\circ$

7a.

$$f(x) = \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{x^2-9}$$

$$= \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{(x+3)(x-3)}$$

$$= \frac{(4x-5)(x+3)}{(2x+1)(x+3)(x-3)} - \frac{2x(2x+1)}{(2x+1)(x+3)(x-3)}$$

$$= \frac{(4x-5)(x+3) - 2x(2x+1)}{(2x+1)(x+3)(x-3)}$$

$$\frac{4x^2+7x-15-4x^2-2x}{(2x+1)(x+3)(x-3)}$$

$$=\frac{5x-15}{(2x+1)(x+3)(x-3)}$$

$$=\frac{5\cancel{(x-3)}}{(2x+1)(x+3)\cancel{(x-3)}}$$

$$=\frac{5}{(2x+1)(x+3)}$$

7b. $f(x) = \dfrac{5}{(2x+1)(x+3)}$ $\quad f = 5$ $\quad g = (2x+1)(x+3)$

$f' = 0$ $\quad = 2x^2+7x+3$

$g' = 4x+7$

$$f'(x) = \frac{0(2x+1)(x+3) - 5(4x+7)}{\left((2x+1)(x+3)\right)^2}$$

$$=\frac{-5(4x+7)}{(2x+1)^2(x+3)^2}$$

at P, $x=-1$, $\quad P\left(-1, -5/2\right)$

$$f'(-1) = -\frac{15}{4}$$

$\therefore$ m of normal $\dfrac{4}{15}$ $\quad$ (since $\perp$)

$$y - -5/2 = \frac{4}{15}(x - -1)$$

$$y + 5/2 = \frac{4}{15}(x+1)$$

8a. $2\cos 3x - 3\sin 3x = R\cos(3x+\alpha)$

$= R\cos 3x\cos\alpha - R\sin 3x\sin\alpha$

$R = \sqrt{2^2+(-3)^2}$

$= \sqrt{13}$

$\cos 3x$: $2 = R\cos\alpha$

$2 = \sqrt{13}\cos\alpha$

$\alpha = \cos^{-1}\left(\frac{2}{\sqrt{13}}\right)$

$= 0.983$ (3 s.f.)

$2\cos 3x - 3\sin 3x = \sqrt{13}\cos(3x + 0.983)$

8b. $f(x) = e^{2x}\cos(3x)$

$f'(x) = 2e^{2x}\cos 3x - 3\sin 3x\, e^{2x}$

$= e^{2x}(2\cos 3x - 3\sin 3x)$

$= e^{2x}.\sqrt{13}\cos(3x + 0.983)$

$= \sqrt{13}\, e^{2x}\cos(3x+0.983)$

8c. turning point occurs when $f'(x) = 0$

$\sqrt{13}\, e^{2x}\cos(3x+0.983) = 0$

$\cos(3x+0.983) = 0$ $\quad (e^{2x} > 0,\ \forall x \in \mathbb{R})$

P.V. $3x + 0.983 = \pi/2$ (smallest positive)

$3x = \pi/2 - 0.983$

$x = \frac{\pi/2 - 0.983}{3} = 0.196$ (3 s.f.)