Edexcel

A Level

A Level Maths

Edexcel Core Maths C2 June 2011 Model Solutions

Name:

M M E

Mathsmadeeasy.co.uk

Total Marks:

Edexcel June 2011 C2

1a. $f(x) = 2x^3 - 7x^2 - 5x + 4$

$f(1) = 2(1)^3 - 7(1)^2 - 5(1) + 4$

$= -6$

1b. $f(-1) = 2(-1)^3 - 7(-1)^2 - 5(-1) + 4$

$= 0 \quad \therefore \quad x+1$ is a factor

1c.

$$
\begin{array}{r|l}
 & 2x^2 - 9x + 4 \\
\hline
x-1 & 2x^3 - 7x^2 - 5x + 4 \\
 & \underline{2x^3 - 2x^2} \\
 & -9x^2 - 5x \\
 & \underline{-9x^2 - 9x} \\
 & 4x + 4 \\
 & \underline{4x + 4} \\
 & 0
\end{array}
$$

$\therefore \quad f(x) = (x-1)(2x^2 - 9x + 4)$

$= (x-1)(2x-1)(x-4)$

2a. $(3 + bx)^5 = {}^5C_0\, 3^5 + {}^5C_1\, 3^4 (bx)^1 + {}^5C_2\, 3^3 (bx)^2 + \ldots$

$= 243 + 405bx + 270b^2x^2 + \ldots$

2b. $270b^2 = 2(405b)$

$270b^2 = 810b$

$270b(b-3) = 0$

$b = 0$ or 3

$\therefore \quad b = 3 \quad$ (since $b \neq 0$)

3a.

$5^x = 10$

$x \log 5 = \log 10$

$x = \dfrac{\log 10}{\log 5}$

$= 1.43 \quad (3sf)$

3b.

$\log_3(x-2) = -1$

$x - 2 = 3^{-1}$

$x = 2 + \frac{1}{3}$

$= 7/3$

4a.

$x^2 + y^2 + 4x - 2y - 11 = 0$

$(x+2)^2 + (y-1)^2 - 4 - 1 - 11 = 0$

$(x+2)^2 + (y-1)^2 = 16$

$\therefore$ Centre $(-2, 1)$

4b.

radius $= \sqrt{16} = 4$

4c.

crosses y axis when $x = 0$

$(0+2)^2 + (y-1)^2 = 16$

$4 + (y-1)^2 = 16$

$(y-1)^2 = 12$

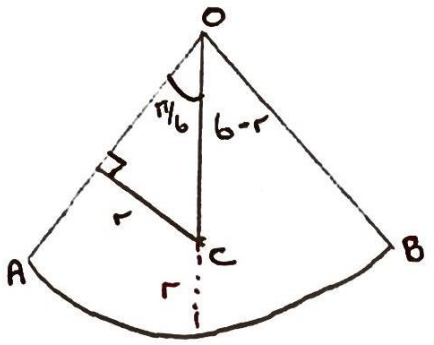
$(y-1) = \pm\sqrt{12}$

$y = 1 \pm \sqrt{12}$

$= 1 \pm 2\sqrt{3}$

5a. $A = \frac{1}{2}r^2\theta \quad ; \quad \frac{1}{2}(6)^2(\pi/3) = 6\pi$

5b.



$\sin\frac{\pi}{6} = \frac{r}{6-r}$

$\frac{1}{2} = \frac{r}{6-r}$

$6-r = 2r$

$3r = 6$

$\therefore r = 2$

5c. A of O $= \pi r^2 = 4\pi$

$\therefore$ Shaded Area : $6\pi - 4\pi = 2\pi$

6a. $u_2 = 192 = ar$ ①

$u_3 = 144 = ar^2$ ②

$U_n = ar^{n-1}$

'② ÷ ①'

$\frac{144}{192} = \frac{ar^2}{ar}$

$3/4 = r$

6b. 'sub $r = 3/4$ into ①'

$192 = a(3/4)$

$a = \frac{192}{(3/4)}$

$= 256$

6c. $S_\infty = \frac{a}{1-r}$

$= \frac{256}{1-3/4} = 1024$

6d.

$$S_n = \frac{a(1-r^n)}{1-r} \qquad a = 256 \qquad r = 3/4$$

$$S_n > 1000$$

$$\frac{256(1-0.75^n)}{1-0.75} > 1000$$

$$256(1-0.75^n) > 250$$

$$1-0.75^n > 125/128$$

$$0.75^n < 3/128$$

$$n \log 0.75 < \log(3/128)$$

$$n > \frac{\log(3/128)}{\log 0.75}$$

Inequality flips since $\log 0.75 < 1$

$$n > 13.047\ldots$$

$$n = 14$$

7a.

$$3\sin(x+45) = 2$$

$$\sin(x+45) = \frac{2}{3}$$

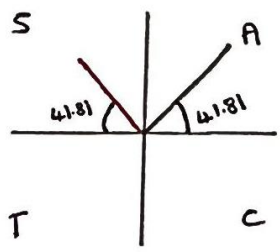
$$0 \leq x < 360$$

$$45 \leq \phi < 405$$

let $\phi = x+45$

$$\sin\phi = 2/3$$

P.V. $\phi = 41.81$



41.81 not in range

$$\phi = 138.2,\ 401.8$$

$$\phi = x+45$$

$$138.2 = x+45 \Rightarrow x = 93.2$$

$$401.8 = x+45 \Rightarrow x = 356.8$$

7b. $2\sin^2 x + 2 = 7\cos x \qquad 0 \leq x < 2\pi$

$2(1-\cos^2 x) + 2 = 7\cos x$

$2 - 2\cos^2 x + 2 = 7\cos x$

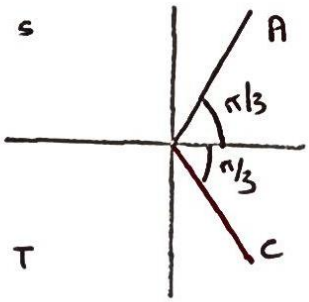
$2\cos^2 x + 7\cos x - 4 = 0$

$(2\cos x - 1)(\cos x + 4) = 0$

$2\cos x = 1$ or $\cos x = -4$

$\cos x = 1/2$ $\qquad$ ✗ no solutions

P.V. $x = \pi/3$



$x = \pi/3,\ 5\pi/3$

8a. Let the depth of cuboid be l

$V = 2x \times x \times l = 81$

$2x^2 l = 81$

$l = \dfrac{81}{x^2}$

$L = 2(2x + x) + 2(2x + x) + 4l$

$= 12x + 4\left(\dfrac{81}{2x^2}\right)$

$= 12x + \dfrac{162}{x^2}$

8b. $L = 12x + 162x^{-2}$

$\frac{dL}{dx} = 12 - 324x^{-3}$

min. when $\frac{dL}{dx} = 0$

$\frac{324}{x^3} = 12$

$x^3 = 27$

$x = 3$

when $x = 3$, $L = 36 + \frac{162}{9}$

$= 54$

8c. $\frac{d^2L}{dx^2} = 972x^{-4}$

when $x = 3$, $\frac{d^2L}{dx^2} = \frac{972}{3^4}$

$= 12$

$\frac{d^2L}{dx^2} > 0$ $\therefore$ minimum point

9a. $y = -x^2 + 2x + 24$, $y = x + 4$

$x + 4 = -x^2 + 2x + 24$

$x^2 - x - 20 = 0$

$(x-5)(x+4) = 0$; $x = -4$ or $x = 5$

A $(-4, 0)$

when $x = 5$, $y = 5 + 4 = 9$

B $(5, 9)$

9b.

$$R = \int_{-4}^{5} -x^2 + 2x + 24 \, dx - \int_{-4}^{5} x + 4 \, dx$$

$$= \left[-\frac{1}{3}x^3 + x^2 + 24x \right]_{-4}^{5} - \left[\frac{1}{2}x^2 + 4x \right]_{-4}^{5}$$

$$= \left(\frac{310}{3} - - \frac{176}{3} \right) - \left(65/2 - - 8 \right)$$

$$= 162 - \frac{81}{2}$$

$$= \frac{243}{2}$$