Edexcel

A Level

A Level Maths

Edexcel Core Maths C4 June 2010 Model Solutions

Name:

MME
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Total Marks:

Edexcel June 10 C4

1a. $x = \pi/6 \quad , \quad y = 1.2247$

$x = \pi/4 \quad , \quad y = 1.1180$

1bi. $h = \pi/6 \Rightarrow R \approx \frac{1}{2}\left(\frac{\pi}{6}\right)\left\{(1.3229 + 1) + 2(1.2247)\right\}$

$= 1.249 \quad (3dp)$

1bii. $h = \pi/12 \Rightarrow R \approx \frac{1}{2}\left(\frac{\pi}{12}\right)\left\{(1.3229 + 1) + 2(1.2973 + 1.2247 + 1.118)\right\}$

$= 1.257 \quad (3dp)$

2. $$\int_0^{\pi/2} e^{\cos x + 1} \sin x \, dx$$

$u = \cos x + 1$

$\frac{du}{dx} = -\sin x$

$dx = \frac{-\sin x}{du}$

$$\int_2^1 e^u \sin x \cdot \frac{du}{-\sin x}$$

$$-\int_2^1 e^u \, du$$

x	$\pi/2$	0
u	1	2

$\left(\begin{array}{l} \cos(\pi/2) + 1 = 1 \\ \cos(0) + 1 = 2 \end{array} \right)$

$$= \int_1^2 e^u \, du$$

$$= \left[e^u\right]_1^2$$

$$= e^2 - e$$

$$= e(e-1)$$

3. $2^x + y^2 = 2xy$

$$\frac{d}{dx}\left(2^x\right) = \frac{d}{dx}\left(e^{\ln 2^x}\right) = \frac{d}{dx}\left(e^{x\ln 2}\right)$$

$$= \ln 2 . e^{x\ln 2}$$

$$= \ln 2 . 2^x$$

$$\frac{d}{dx}\left(y^2\right) = 2y . \frac{dy}{dx}$$

$$\frac{d}{dx}\left(2xy\right) = 2y + 2x . \frac{dy}{dx}$$

$\therefore$

$$\ln 2 . 2^x + 2y\frac{dy}{dx} = 2y + 2x . \frac{dy}{dx}$$

$$2^x \ln 2 - 2y = \frac{dy}{dx}\left(2x - 2y\right)$$

$$\frac{dy}{dx} = \frac{2^x \ln 2 - 2y}{2x - 2y}$$

at $(3,2)$, $$\frac{dy}{dx} = \frac{2^3 \ln 2 - 2(2)}{2(3) - 2(2)}$$

$$= \frac{8\ln 2 - 4}{2}$$

$$= 4\ln 2 - 2$$

4a

$x = \sin^2 t$, $y = 2\tan t$, $0 \leq t < \frac{\pi}{2}$

$x = (\sin t)^2$

$\frac{dy}{dt} = 2\sec^2 t$

$\frac{dx}{dt} = 2\cos t \sin t$

$= \frac{2}{\cos^2 t}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{2}{\cos^2 t} \times \frac{1}{2\cos t \sin t}$

$= \frac{1}{\cos^3 t \sin t}$

4b

when $t = \pi/3$ $x = \sin^2(\pi/3) = 3/4$

$y = 2\tan(\pi/3) = 2\sqrt{3}$

so point at $(3/4, 2\sqrt{3})$

when $t = \pi/3$, $\frac{dy}{dx} = \frac{1}{\cos^3(\pi/3)\sin(\pi/3)}$

$= \frac{16}{\sqrt{3}}$

$y - 2\sqrt{3} = \frac{16}{\sqrt{3}}(x - 3/4)$

crosses x axis, when $y = 0$

$0 - 2\sqrt{3} = \frac{16}{\sqrt{3}}(x - 3/4)$

$x = \frac{-2\sqrt{3}.\sqrt{3}}{16} + 3/4$

$= 3/8$

5a.

$$\frac{2x^2+5x-10}{(x-1)(x+2)} = A + \frac{B}{x-1} + \frac{C}{x+2}$$

$$2x^2+5x-10 = A(x-1)(x+2) + B(x+2) + C(x-1)$$

when $x = 1$; $-3 = 3B \Rightarrow B = -1$

when $x = -2$; $-12 = -3C \Rightarrow C = 4$

when $x = 0$; $-10 = -2A + 2B - C$

$-10 = -2A - 2 - 4$

$-4 = -2A \Rightarrow A = 2$

5b.

$$\frac{2x^2+5x-10}{(x-1)(x+2)} = 2 - \frac{1}{x-1} + \frac{4}{x+2}$$

$$= 2 - (x-1)^{-1} + 4(x+2)^{-1}$$

$$= 2 + (1-x)^{-1} + 4(2+x)^{-1} \quad (*)$$

$$(1-x)^{-1} = 1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2 + \dots$$

$$= 1 + x + x^2 + \dots \quad ①$$

$$4(2+x)^{-1} = 4\left[2(1+x/2)\right]^{-1} = \frac{4}{2}(1+x/2)^{-1}$$

$$= 2\left(1 + (-1)(x/2) + \frac{(-1)(-2)}{2}(x/2)^2 + \dots\right)$$

$$= 2(1 - x/2 + x^2/4)$$

$$= 2 - x + \frac{x^2}{2} \quad ②$$

'Sub ① and ② into (*)'

$$2 + (1 + x + x^2) + (2 - x + x^2/2)$$

$$= 5 + 3/2\, x^2$$

6a. $$4\cos^2\theta - 3\sin^2\theta \equiv \frac{1}{2} + \frac{7}{2}\cos 2\theta$$

LHS $\quad 4\cos^2\theta - 3\sin^2\theta$

use $\cos^2\theta \equiv \frac{1}{2}(1 + \cos 2\theta)$

$\sin^2\theta \equiv \frac{1}{2}(1 - \cos 2\theta)$

$$= 4 \cdot \frac{1}{2}(1 + \cos 2\theta) - 3 \cdot \frac{1}{2}(1 - \cos 2\theta)$$

$$= 2 + 2\cos 2\theta - \frac{3}{2} + \frac{3}{2}\cos 2\theta$$

$$= \frac{1}{2} + \frac{7}{2}\cos 2\theta \quad = \quad \text{RHS}$$

6b. $$\int_0^{\pi/2} \theta\left(\frac{1}{2} + \frac{7}{2}\cos 2\theta\right) d\theta$$

$$= \int_0^{\pi/2} \frac{1}{2}\theta \, d\theta + \frac{7}{2}\underbrace{\int_0^{\pi/2} \theta \cos 2\theta \, d\theta} \qquad (+)$$

$$\int_0^{\pi/2} \theta \cos 2\theta \, d\theta \qquad \text{Parts}$$

$u = \theta \qquad v' = \cos 2\theta$

$u' = 1 \qquad v = \frac{1}{2}\sin 2\theta$

$$= \left[\frac{1}{2}\theta \sin 2\theta\right]_0^{\pi/2} - \int \frac{1}{2}\sin 2\theta \, d\theta$$

$$= \left[\frac{1}{2}\theta \sin 2\theta + \frac{1}{4}\cos 2\theta\right]_0^{\pi/2} \qquad ①$$

'Sub ① into (+)'

$$\int_0^{\pi/2} \tfrac{1}{2}\theta \, d\theta + \tfrac{7}{2}\left[\tfrac{1}{2}\theta \sin 2\theta + \tfrac{1}{4}\cos 2\theta\right]_0^{\pi/2}$$

$$= \left[\tfrac{1}{4}\theta^2 + \tfrac{7}{2}\left(\tfrac{1}{2}\theta \sin 2\theta + \tfrac{1}{4}\cos 2\theta\right)\right]_0^{\pi/2}$$

$$= \left[\tfrac{1}{4}\theta^2 + \tfrac{7}{4}\theta \sin 2\theta + \tfrac{7}{8}\cos 2\theta\right]_0^{\pi/2}$$

$$= \left(\tfrac{1}{4}\left(\tfrac{\pi}{2}\right)^2 + 0 - 7/8\right) - \left(0 + 0 + 7/8\right)$$

$$= \tfrac{1}{4} \cdot \tfrac{\pi^2}{4} - 7/4$$

$$= \tfrac{1}{16}\pi^2 - 7/4$$

7a.

$$l_1 : \quad \underline{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$l_2 : \quad \underline{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$$

$l_1 = l_2 \Rightarrow$

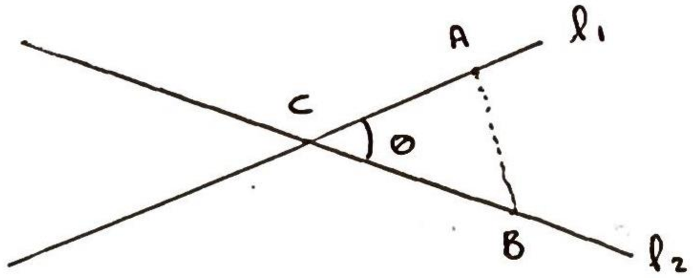
$$2 + \lambda = 0 + 5\mu \quad ①$$
$$3 + 2\lambda = 9 \quad ②$$
$$-4 + \lambda = -3\mu + 2 \quad ③$$

from ② $\lambda = 3$

sub in ① $2 + 3 = 5\mu \Rightarrow \mu = 1$

$$\lambda = 3 \Rightarrow \underline{r} = \begin{pmatrix} 2+3 \\ 3+2(3) \\ -4+3 \end{pmatrix} = (5, 9, -1)$$

7b.



$C(5, 9, -1)$

$$\cos\theta = \frac{\overrightarrow{CA} \cdot \overrightarrow{CB}}{|CA|\ |CB|}$$

when $\lambda = 0$ $\quad A = (2, 3, -4)$

when $m = -1$ $\quad B = (-5, 9, -5)$

$$\overrightarrow{CA} = \begin{pmatrix} -3 \\ -6 \\ -3 \end{pmatrix} \qquad |CA| = \sqrt{3^2 + 6^2 + 3^2} = \sqrt{54}$$

$$\overrightarrow{CB} = \begin{pmatrix} -10 \\ 0 \\ -4 \end{pmatrix} \qquad |CB| = \sqrt{10^2 + 0^2 + 4^2} = \sqrt{116}$$

$$\cos\theta = \frac{(-3)(-10) + (-6)0 + (-3)(-4)}{\sqrt{54} \times \sqrt{116}}$$

$$= \frac{42}{6\sqrt{174}}$$

$$\theta = 57.95^\circ \quad (2dp)$$

7c.

$$A = \frac{1}{2}|AC| \times |BC| \times \sin(57.95^\circ)$$

$$= 33.5 \quad (3sf)$$

8a.

$$\frac{dV}{dt} = 0.48\pi - 0.6\pi h$$

we want $\frac{dh}{dt}$; $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$

$V = \pi r^2 h = \pi . 9h \quad (r=3)$

$\frac{dV}{dh} = 9\pi \quad \Rightarrow \quad \frac{dh}{dV} = \frac{1}{9\pi}$

$$\frac{dh}{dt} = (0.48\pi - 0.6\pi h) \times \frac{1}{9\pi}$$

$$= \frac{4}{75} - \frac{h}{15}$$

$$75\frac{dh}{dt} = 4 - 5h$$

8b.

$$\int \frac{75}{4-5h}\, dh = \int 1 \, dt$$

$$-15\int \frac{-5}{4-5h}\, dh = \int 1 \, dt$$

$$-15 \ln|4-5h| = t + c$$

when $t=0$, $h=0.2$: $-15\ln|3| = 0 + c$

$c = -15\ln 3$

$$-15\ln|4-5h| = t - 15\ln 3$$

when $h = 0.5$, $-15\ln|3/2| = t - 15\ln 3$

$t = -15\ln|3/2| + 15\ln 3$

$= 10.4 \quad (3sf)$