Edexcel

A Level

A Level Maths

Edexcel Core Maths C3 June 2010 Model Solutions

Name:

M M E

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Total Marks:

Edexcel June 10 C3

1a.

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$$

$$= \frac{2\sin\theta\cos\theta}{1+(2\cos^2\theta - 1)}$$

$\sin 2\theta \equiv 2\sin\theta\cos\theta$

$\cos 2\theta \equiv 2\cos^2\theta - 1$

$$= \frac{2\sin\theta\cos\theta}{2\cos^2\theta}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$

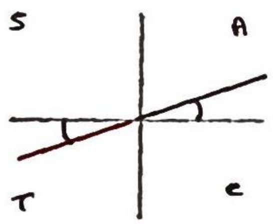
1b.

$2\tan\theta = 1$

$-180° \leq \theta < 180$

$\tan\theta = 1/2$

P.V. $\theta = 26.565°$



$\theta = 26.6°, -153.4°$

2.

$$y = \frac{3}{(5-3x)^2}$$

Quotient Rule

$f = 3$

$f' = 0$

$g = (5-3x)^2$

$g' = 2(-3)(5-3x)^1$

$g' = -6(5-3x)$

$$\frac{dy}{dx} = \frac{0 - 18(5-3x)}{((5-3x)^2)^2}$$

$$= \frac{-18(5-3x)}{(5-3x)^4}$$

$= \dfrac{-18}{(5-3x)^3}$

when $x = 2$, $y = \dfrac{3}{(5-3(2))^2} = 3$, $C\ (2, 3)$

when $x = 2$, $\dfrac{dy}{dx} = \dfrac{-18}{(5-3(2))^3} = -18$

$\therefore$ m of normal $= \dfrac{1}{18}$ (Since $\perp$)

$y - 3 = \dfrac{1}{18}(x-2)$

$18y - 54 = x - 2$

$x - 18y + 52 = 0$

3a. $f(x) = 4\operatorname{cosec} x - 4x + 1$

$f(1.2) = 0.49166\ldots$

$f(1.3) = -0.0487\ldots$

change of sign $\therefore$ $1.2 < \alpha < 1.3$

3b. $4\operatorname{cosec} x - 4x + 1 = 0$

$\dfrac{4}{\sin x} + 1 = 4x$

$x = \dfrac{1}{\sin x} + \dfrac{1}{4}$

3c. $x_{n+1} = \dfrac{1}{\sin x_n} + \dfrac{1}{4}$

$x_0 = 1.25$

$x_1 = 1.3038$

$x_2 = 1.2867$

$x_3 = 1.2917$

3d. $\alpha = 1.291 \iff 1.2905 \leq \alpha < 1.2915$

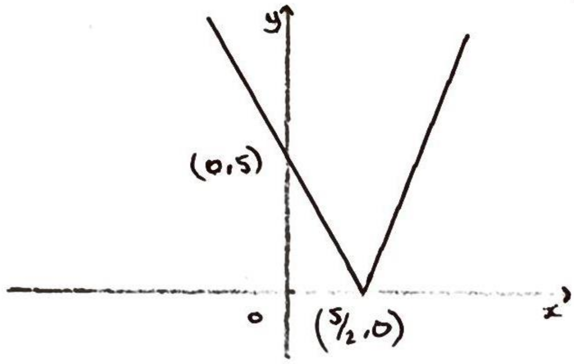
$f(1.2905) = 0.000045\ldots$

$f(1.2915) = -0.0048\ldots$

change of sign $\Rightarrow 1.2905 < \alpha < 1.2915$

$\therefore \alpha = 1.291$ (3dp)

4a.



$f(x) = |2x-5|$

4b.

$f(x) = 15 + x$

$|2x-5| = 15+x$

$(2x-5)^2 = (15+x)^2$

$4x^2 - 20x + 25 = x^2 + 30x + 225$

$3x^2 - 50x - 200 = 0$

$(3x+10)(x-20) = 0$

$x = 20$ or $x = -10/3$

4c.

$g(x) = x^2 - 4x + 1 \quad x \in \mathbb{R}, \ 0 \leq x \leq 5$

$fg(x)$;

$g(2) = 2^2 - 4(2) + 1$

$= -3$

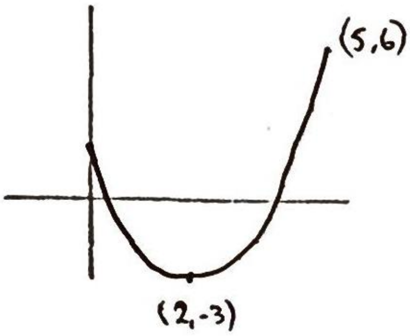
$fg(2) = f(-3) = |2(-3) - 5|$

$= 11$

4d. $g(x) = x^2 - 4x + 1 \qquad 0 \leq x \leq 5$

$= (x-2)^2 - 4 + 1$

$= (x-2)^2 - 3$



$\therefore \quad -3 \leq g(x) \leq 6$

5a. $y = (2x^2 - 5x + 2)e^{-x}$

crosses y axis when $x = 0$;

$y = (2(0)^2 - 5(0) + 2)e^{-0}$

$= 2$

$\therefore \quad (0, 2)$

5b. $(2x^2 - 5x + 2)e^{-x} = 0$

since $e^{-x} > 0 \ \forall x$, can divide through by e^{-x}

$2x^2 - 5x + 2 = 0$

$(2x-1)(x-2) = 0$

$x = 1/2$ and $x = 2$

5c. $y = (2x^2 - 5x + 2)e^{-x}$

Product Rule

$f = 2x^2 - 5x + 2 \qquad g = e^{-x}$

$f' = 4x - 5 \qquad g' = -e^{-x}$

$\frac{dy}{dx} = -e^{-x}(2x^2 - 5x + 2) + e^{-x}(4x - 5)$

5d. At turning points, $\frac{dy}{dx} = 0$

$$-e^{-x}(2x^2-5x+2) + e^{-x}(4x-5) = 0.$$

$$-(2x^2-5x+2) + 4x-5 = 0 \qquad \div e^{-x} \quad (e^{-x} > 0 \ \forall x \in \mathbb{R})$$

$$-2x^2+5x-2+4x-5=0$$

$$2x^2-9x+7=0$$

$$(2x-7)(x-1)=0$$

$$x=1 \quad \text{or} \quad x=7/2$$

when $x=1$, $y = (2(1)^2-5(1)+2)e^{-1}$

$= e^{-1}$

$(1, e^{-1})$

when $x=7/2$ $y = (2(7/2)^2-5(7/2)+2)e^{-7/2}$

$= 9e^{-7/2}$

$(7/2, 9e^{-7/2})$

6ai. A $(3,-4)$

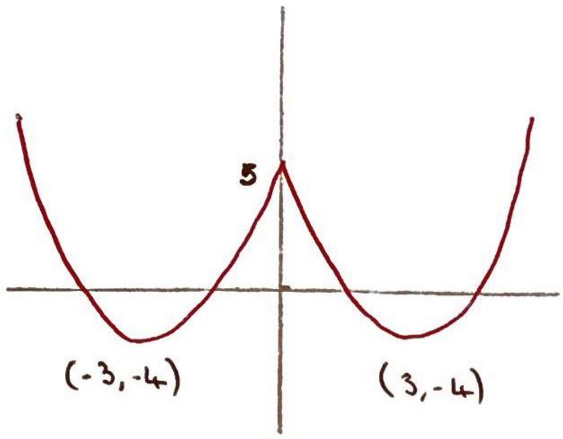
$y = |f(x)|$ A $(3,4)$

6aii $y = 2f(\frac{1}{2}x)$ stretch s.f. 2 in y direction (y's double)

stretch s.f. $1/\frac{1}{2}$ in x direction (x's double)

A $(6,-8)$

6b.



$y = f|x|$

6c.

Translation 3 right, 4 down

$\therefore \quad f(x-3) - 4$

$= (x-3)^2 - 4$

6d.

$f(x)$ is not one to one $\therefore$ doesn't have an inverse

7a.

$2\sin\theta - 1.5\cos\theta = R\sin(\theta - \alpha)$

$= R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$

$R = \sqrt{2^2 + 1.5^2}$

$= 5/2$

$\sin\theta$; $\quad 2 = R\cos\alpha$

$\alpha = \cos^{-1}\left(\frac{2}{5/2}\right)$

$= 0.6435$ (4dp)

$2\sin\theta - 1.5\cos\theta = \frac{5}{2}\sin(\theta - 0.6435)$

7bi.

max of $\sin(\theta - 0.6435) = 1$

$\therefore \quad \frac{5}{2} \times 1 = \frac{5}{2}$

7bii.

$$\sin(\theta - 0.6435) = 1 \qquad 0 \leq \theta < \pi$$

$$\theta - 0.6435 = \pi/2$$

$$\theta = 2.21 \quad (2dp)$$

7ci.

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right)$$

$$= 6 + \frac{5}{2}\sin\left(\frac{4\pi t}{25} - 0.6435\right)$$

max when $\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1$

$$= 6 + \frac{5}{2} \times 1$$

$$= \frac{17}{2}$$

7cii.

from bii) max when $\theta = 2.214$

$$\frac{4\pi t}{25} = 2.214$$

$$t = \frac{2.214 \times 25}{4\pi}$$

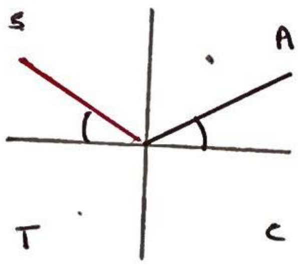
$$t = 4.41 \quad (3sf)$$

7d.

$$7 = 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right)$$

$$\sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{2}{5}$$

$$\arcsin\left(2/5\right) = 0.4115,\ 2.73$$



$$\frac{4\pi t}{25} - 0.6435 = 0.4115$$

$$t = \frac{(0.4115 + 0.6435) \times 25}{4\pi}$$

$$= 2.0988\ldots \text{ hours}$$

$$= 2 \text{ hours } 6 \text{ mins}$$

so 14:06

or $\frac{4\pi t}{25} - 0.6435 = 2.73$

$$t = \frac{(2.73 + 0.6435) \times 25}{4\pi}$$

$$= 6.71136\ldots \text{ hours}$$

$$= 6 \text{ hours } 43 \text{ mins}$$

so 18:43

8a.

$$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}$$

$$= \frac{(2x-1)(x+5)}{(x+5)(x-3)}$$

$$= \frac{2x-1}{x-3}$$

8b.

$$\ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15)$$

$$\ln\left(\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}\right) = 1$$

$$\ln\left(\frac{2x - 1}{x - 3}\right) = 1$$

$$\frac{2x - 1}{x - 3} = e$$

$$2x - 1 = ex - 3e$$

$$2x - ex = 1 - 3e$$

$$x(2 - e) = 1 - 3e$$

$$x = \frac{1 - 3e}{2 - e}$$