Edexcel

A Level

A Level Maths

Edexcel Core Maths C3 January 2013 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

Edexcel Jan 13 C3

1a. $y = (2x-3)^5$

$-32 = (2x-3)^5 \Rightarrow 2x-3 = -2$

$x = 1/2 = \omega$

1b. $\frac{dy}{dx} = 5.2\,(2x-3)^4$

$= 10(2x-3)^4$

at P $\frac{dy}{dx} = 10(2(1/2)-3)^4 = 160$

$\Rightarrow y+32 = 160(x-1/2)$

$y = 160x - 112$

2a. $g(x) = e^{x-1} + x - 6 = 0$

$e^{x-1} = 6-x$

$x-1 = \ln|6-x|$

$x = \ln|6-x| + 1$

2b. $x_{n+1} = \ln(6-x_n) + 1$

$x_0 = 2$, $x_1 = 2.3863$, $x_2 = 2.2847$, $x_3 = 2.3125$

2c. $\alpha \in [2.3065, 2.3075)$

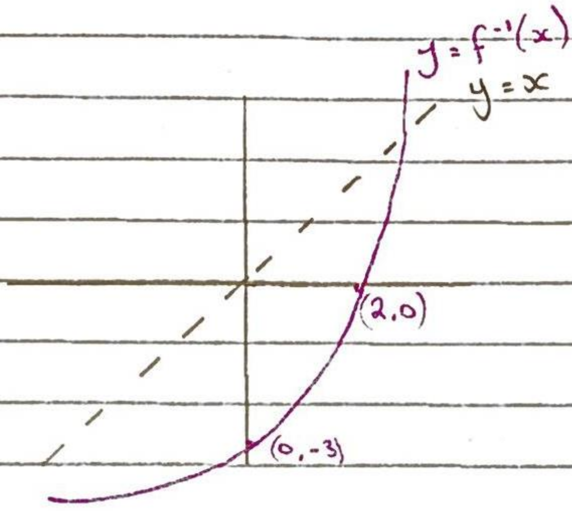
$g(2.3065) = -0.000275$,

$g(2.3075) = 0.00442$

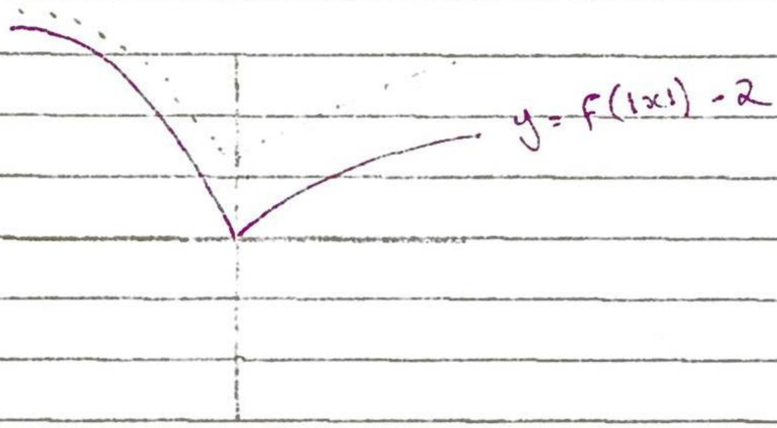
change of sign $\Rightarrow \alpha \in (2.3065, 2.3075) \Rightarrow \alpha = 2.307$ to 3dp

3a $f(-3) = 0$, $ff(-3) = f(0) = 2$

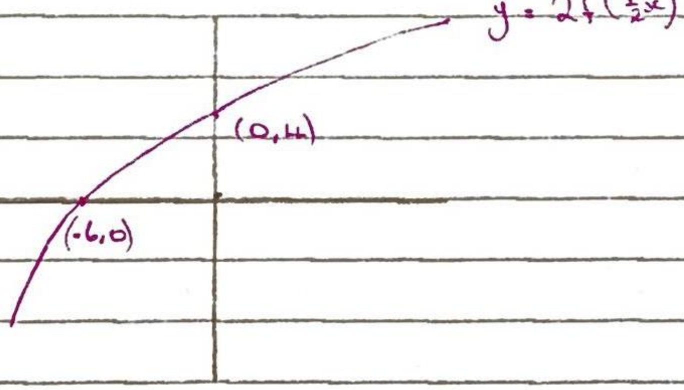
3b.



3c.



3c.



4a $6\cos\theta + 8\sin\theta = R\cos(\theta - \alpha)$

$6\cos\theta + 8\sin\theta = R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$

$R^2 = 6^2 + 8^2$; $R = 10$

$6\cos\theta = 10\cos\theta\cos\alpha$ $\alpha = \cos^{-1}(6/10) = 0.927$

4bi. $P(\theta) = 4(12 + 6\cos\theta + 8\sin\theta)^{-1}$ $0 \leq \theta \leq 2\pi$

$$P(\theta) = \frac{4}{10\cos(\theta - 0.927) + 12}$$

$$\text{max}: \frac{4}{12 - 10} = 2$$

4bii. max when $\cos(\theta - 0.927) = -1$

$$\theta - 0.927 = \pi$$

$$\theta = 4.07^c$$

5ai. $\frac{d}{dx}\left[x^3 \ln 2x\right] = 3x^2 \ln 2x + \frac{2x^3}{2x}$

$= 3x^2 \ln 2x + x^2$

5bi. $\frac{d}{dx}\left[(x+\sin 2x)^3\right] = 3(1+2\cos 2x)(x+\sin 2x)^2$

5ii. $x = \cot y \quad , \quad x^2 = \cot^2 y$

$\frac{dx}{dy} = -\operatorname{cosec}^2 y \quad \Rightarrow \frac{dy}{dx} = -\frac{1}{\operatorname{cosec}^2 y}$

$\operatorname{cosec}^2 y = 1 + \cot^2 y = 1 + x^2$

so $\frac{dy}{dx} = -\frac{1}{1+x^2}$

6i. $(\sin 22.5 + \cos 22.5)^2$

$= \sin^2 22.5 + 2\sin 22.5 \cos 22.5 + \cos^2 22.5$

$(\sin^2 22.5 + \cos^2 22.5 = 1)$

$(2\sin 22.5 \cos 22.5 = \sin 45°)$

$1 + \sin 45 = 1 + \frac{\sqrt{2}}{2}$

6iia. $\cos 2\theta + \sin\theta = 1$

use : $\cos 2\theta \equiv 1 - 2\sin^2\theta \quad \forall\, \theta \in \mathbb{R}$

$1 - 2\sin^2\theta + \sin\theta - 1 = 0$

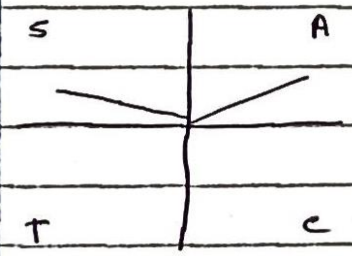
$2\sin^2\theta - \sin\theta = 0 \quad , \quad k = 2$

6b. $\sin\theta(2\sin\theta - 1) = 0 \qquad 0 \leq \theta < 360°$

$\sin\theta = 0 \quad \Rightarrow \quad \theta = 0°, 180°$

$2\sin\theta = 1$

$\sin\theta = 1/2 \quad \Rightarrow \quad \theta = 30°, 150°$



7a. $h(x) = \dfrac{2}{x+2} + \dfrac{4}{x^2+5} - \dfrac{18}{(x^2+5)(x+2)}$

$= \dfrac{1}{(x^2+5)(x+2)} \cdot \left(2(x^2+5) + 4(x+2) - 18\right)$

$= \dfrac{2x^2 + 10 + 4x + 8 - 18}{}$

$\dfrac{2(x^2+2x)}{(x^2+5)(x+2)} = \dfrac{2x}{x^2+5}$

7b. $f = 2x \qquad g = x^2+5$

$f' = 2 \qquad g' = 2x$

$h'(x) = \dfrac{2(x^2+5) - 2x(2x)}{(x^2+5)^2}$

$= \dfrac{-2x^2+10}{(x^2+5)^2}$

7c. max value of $h(x)$ at max point

$h'(x) = 0 \Rightarrow 2x^2 = 10$

$x = \pm\sqrt{5}$ ($+\sqrt{5}$ from diagram)

$h(\sqrt{5}) = \dfrac{\sqrt{5}}{5}$ so max point $\left(\sqrt{5}, \dfrac{\sqrt{5}}{5}\right)$

so $0 \leq h(x) \leq \dfrac{\sqrt{5}}{5}$

8a $V = 17{,}000e^{-0.25t} + 2000e^{-0.5t} + 500$

$t = 0 \Rightarrow V = 19{,}500$

8b. $V = 9{,}500$

$\therefore 17{,}000e^{-t/4} + 2000e^{-t/2} - 9000 = 0$

$17{,}000e^{t/4} + 2000 - 9000e^{t/2} = 0$

$17e^{t/4} - 9e^{t/2} + 2 = 0$

Let $e^{t/4} = x$, $x^2 = e^{t/2}$

$\therefore 9x^2 - 17x - 2 = 0$

$(9x+1)(x-2) = 0$

$e^{t/4} = -1/9$ or $e^{t/4} = 2$

but $e^x > 0 \ \forall x \in \mathbb{R}$ so impossible

$\frac{t}{4} = \ln 2$

$t = 4\ln 2$

8c. $\frac{dV}{dt} = -4250e^{-0.25t} - 1000e^{-0.5t}$

$t = 8$, $\frac{dV}{dt} = -593.490\ldots$

$\Rightarrow$ decreasing at £593 p/a