Edexcel

A Level

A Level Maths

Edexcel Core Maths C4 January 2012 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

Edexcel Jan 12 C4

1a.

$$2x + 3y^2 + 3x^2y = 4x^2$$

$$2 + 6y\frac{dy}{dx} + 6xy + 3x^2\frac{dy}{dx} = 8x$$

$$\frac{dy}{dx}\left(6y + 3x^2\right) = 8x - 6xy - 2$$

$$\frac{dy}{dx} = \frac{8x - 6xy - 2}{6y + 3x^2}$$

when $x = -1$, $y = 1$

$$\frac{dy}{dx} = \frac{8(-1) - 6(-1)(1) - 2}{6(1) + 3(-1)^2}$$

$$= -\frac{4}{9}$$

1b.

grad of normal $= \frac{9}{4}$ (since $\perp$)

$$y - 1 = \frac{9}{4}(x - -1)$$

$$4y - 4 = 9x + 9$$

$$9x - 4y + 13 = 0$$

2a.

$$\int \cancel{x^2\cos 3x}\ dx$$

$x \sin 3x$

Parts: $u = x$, $u' = 1$; $v' = \sin 3x$, $v = -\frac{1}{3}\cos 3x$

$$= -\frac{1}{3}\cos 3x - \int -\frac{1}{3}\cos 3x \ dx$$

$$= -\frac{1}{3}\cos 3x + \frac{1}{9}\sin 3x + c$$

2b. $\int x^2 \cos 3x \; dx$ Parts: $u = x^2$, $u' = 2x$, $v' = \cos 3x$, $v = \frac{1}{3}\sin 3x$

$$= \frac{1}{3}x^2 \sin 3x - \int \frac{2x}{3} \sin 3x \; dx$$

$$= \frac{1}{3}x^2 \sin 3x - \frac{2}{3}\int x \sin 3x \; dx$$

$$= \frac{1}{3}x^2 \sin 3x - \frac{2}{3}\left\{ \frac{1}{9}\sin 3x - \frac{1}{3}x\cos 3x \right\} + c$$

$$= \frac{1}{3}x^2 \sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x + c$$

3a.

$$(2-5x)^{-2} = \left[2(1-5/2x)\right]^{-2}$$

$$= 2^{-2}(1-5/2x)^{-2}$$

$$= \frac{1}{4}(1-5/2x)^{-2}$$

$$= \frac{1}{4}\left(1 + (-2)(-5/2x) + \frac{(-2)(-3)}{2}(-5/2x)^2 + \dots\right)$$

$$= \frac{1}{4}\left(1 + 5x + \frac{75}{4}x^2 + \dots\right)$$

$$= \frac{1}{4} + \frac{5}{4}x + \frac{75}{16}x^2 + \dots$$

3b.

$$(2+kx)\left(\frac{1}{4} + \frac{5}{4}x + \frac{75}{16}x^2 + \dots\right) = \frac{1}{2} + \frac{7}{4}x + Ax^2 + \dots$$

x coefficients : $2\left(\frac{5}{4}x\right) + \frac{1}{4}kx = \frac{7}{4}x$

$$\frac{5}{2} + \frac{k}{4} = \frac{7}{4}$$

$$10 + k = 7 \quad \Rightarrow \quad k = -3$$

3c. x^2 coefficients : $2\left(\frac{75}{16}x^2\right) + kx\left(\frac{5}{4}x\right) = Ax^2$

$$\frac{75}{8} + \frac{5}{4}\times(-3) = A \qquad A = 45/8$$

4. $V = \pi \int_0^2 y^2 \, dx$ $\qquad y = \sqrt{\dfrac{2x}{3x^2+4}}$

$y^2 = \dfrac{2x}{3x^2+4}$

$V = \pi \int_0^2 \dfrac{2x}{3x^2+4} \, dx$

$= \dfrac{\pi}{3} \int_0^2 \dfrac{6x}{3x^2+4} \, dx$

$= \dfrac{\pi}{3} \left[\ln|3x^2+4| \right]_0^2$

$= \dfrac{\pi}{3} \left(\ln 16 - \ln 4 \right)$

$= \dfrac{\pi}{3} \left(2\ln 4 - \ln 4 \right)$

$= \dfrac{\pi}{3} \ln 4$

$= \dfrac{2\pi}{3} \ln 2$

5a. $x = 4\sin\left(t + \frac{\pi}{6}\right)$ $\qquad y = 3\cos 2t \qquad 0 \leq t < 2\pi$

$\dfrac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right)$ $\qquad \dfrac{dy}{dt} = -6\sin 2t$

$\dfrac{dy}{dx} = \dfrac{-6\sin 2t}{4\cos(t + \pi/6)} = -\dfrac{3\sin 2t}{2\cos(t + \pi/6)}$

5b. $\dfrac{dy}{dx} = 0 \quad \therefore \quad -3\sin 2t = 0$

$\sin 2t = 0 \qquad 0 \leq t < 2\pi$

$2t = 0, \pi, 2\pi, 3\pi$

$t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

when $t=0$, $x = 4\sin\left(0+\frac{\pi}{6}\right)$ $= 2$; $y = 3\cos 0$ $= 3$; $(2,3)$

$t = \pi/2$, $x = 4\sin\left(\frac{\pi}{2}+\frac{\pi}{6}\right)$ $= 2\sqrt{3}$; $y = 3\cos\pi$ $= -3$; $(2\sqrt{3}, -3)$

$t = \pi$, $x = 4\sin(\pi + \pi/6)$ $= -2$; $y = 3\cos 2\pi$ $= 3$; $(-2, 3)$

$t = \frac{3\pi}{2}$ $x = 4\sin\left(\frac{3\pi}{2}+\frac{\pi}{6}\right)$ $= -2\sqrt{3}$; $y = 3\cos 3\pi$ $= -3$; $(-2\sqrt{3}, -3)$

6a. when $x = \frac{\pi}{8}$, $y = 0.73508$

6b. $h = \frac{\pi}{8}$ $\int \approx \frac{1}{2}\left(\frac{\pi}{8}\right)\left\{(0+0) + 2\left(0.73508 + 1.17157 + 1.02280\right)\right\}$

$= 1.1504$ (4dp)

6c. $\int \frac{2\sin 2x}{1+\cos x}\, dx$

$u = 1+\cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$2\sin 2x = 4\cos x\sin x$

$= 4\sin x(u-1)$

$= \int \frac{4\cos x\sin x}{u} \cdot \frac{du}{-\sin x}$

$= -4\int \frac{\cos x}{u} \cdot du$

$= -4\int \frac{u-1}{u}\, du$

$= -4\int 1 - \frac{1}{u}\, du$

$= -4\left[u - \ln u\right] + c$

$$= -4\left(1+\cos x - \ln|1+\cos x|\right) + c$$

$$= 4\ln|1+\cos x| - 4\cos x - 4 + c$$

$$= 4\ln|1+\cos x| - 4\cos x + k \qquad (k = c-4)$$

6d. $$\left[4\ln|1+\cos x| - 4\cos x\right]_0^{\pi/2}$$

$$= (4\ln 1 - 0) - (4\ln 2 - 4)$$

$$= 4\ln 2 - 4$$

$$\text{Error} = \left|(4\ln 2 - 4) - 1.1504\right|$$

$$= 0.077 \quad (2\text{s.f})$$

7a. $$A\begin{pmatrix}2\\-1\\5\end{pmatrix} \qquad B\begin{pmatrix}5\\2\\10\end{pmatrix} \qquad D\begin{pmatrix}-1\\1\\4\end{pmatrix}$$

$$\overrightarrow{AB} = \begin{pmatrix}3\\3\\5\end{pmatrix}$$

7b. $$l: \quad \underline{r} = \begin{pmatrix}2\\-1\\5\end{pmatrix} + \lambda\begin{pmatrix}3\\3\\5\end{pmatrix}$$

7c. $$\frac{\overrightarrow{AB}\cdot\overrightarrow{AD}}{|AB|\,|AD|} = \cos\theta$$

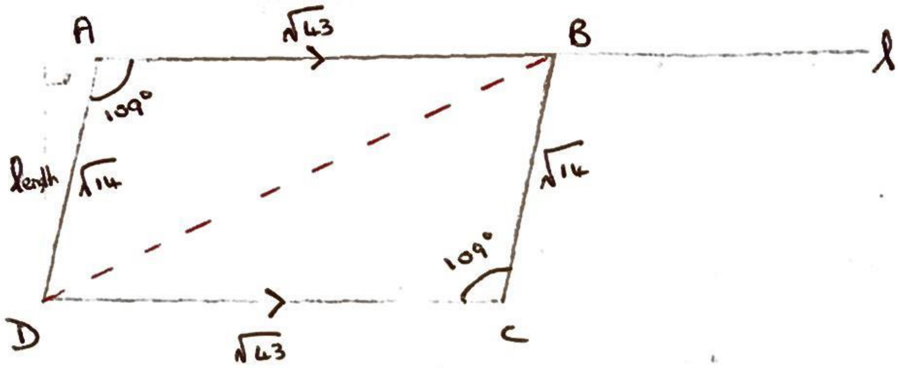
$$\overrightarrow{AD} = \begin{pmatrix}-3\\2\\-1\end{pmatrix}$$

$$\cos\theta = \frac{3(-3)+3(2)+5(-1)}{\sqrt{3^2+3^2+5^2}\cdot\sqrt{3^2+2^2+1^2}}$$

$$= \frac{-8}{\sqrt{602}}$$

$$\theta = 109.0295\ldots = 109^\circ \quad (\text{nearest degree})$$

7d.



$\overrightarrow{DC} = \overrightarrow{AB}$

$\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{AB}$

$= \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} = (2, 4, 9)$

7e.

Area of $\triangle$ DAB $= \frac{1}{2} ab \sin C$

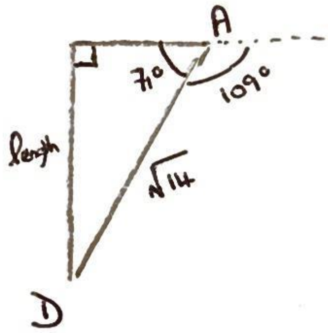
$= \frac{1}{2} \cdot \sqrt{14} \times \sqrt{43} \sin 109^\circ$

Area of parralelogram $= 2 \triangle DAB$

$= \sqrt{14} \times \sqrt{43} \sin 109^\circ$

$= 23.2$ (3 s.f.)

7f.



$\sin 71^\circ = \dfrac{\text{length}}{\sqrt{14}}$

length $= \sqrt{14} \sin 71$

$= 3.54$ (3 s.f.)

8a.

$$\frac{1}{P(5-P)} = \frac{A}{P} + \frac{B}{5-P}$$

$$1 = A(5-P) + BP$$

$P=0$; $1 = 5A \Rightarrow A = 1/5$

$P=5$; $1 = 5B \Rightarrow B = 1/5$

$$\frac{1}{P(5-P)} = \frac{1}{5P} + \frac{1}{5(5-P)}$$

8b.

$$\frac{dP}{dt} = \frac{1}{15} P(5-P) \qquad t \geq 0$$

$$\int \frac{1}{P(5-P)}\, dP = \int \frac{1}{15}\, dt$$

$$\frac{1}{5} \int \frac{1}{P} + \frac{1}{5-P}\, dP = \frac{t}{15} + c$$

$$\frac{1}{5}\left(\ln P - \ln|5-P| \right) = \frac{t}{15} + c$$

when $t=0$, $P=1$

$$\frac{1}{5}\ln 1 - \frac{1}{5}\ln 4 = c \Rightarrow c = -\frac{1}{5}\ln 4$$

$$\frac{1}{5}\left(\ln P - \ln|5-P|\right) = \frac{t}{15} - \frac{1}{5}\ln 4 \qquad (\times 5)$$

$$\ln\left(\frac{P}{5-P}\right) = \frac{t}{3} - \ln 4 \qquad (+\ln 4)$$

$$\ln\left(\frac{4P}{5-P}\right) = \frac{t}{3}$$

$$\frac{4P}{5-P} = e^{\frac{1}{3}t}$$

$$4P = (5-P)e^{1/3 t}$$

$$4P = 5e^{\frac{1}{3}t} - Pe^{\frac{1}{3}t}$$

$$P(4 + e^{\frac{1}{3}t}) = 5e^{\frac{1}{3}t}$$

$$P = \frac{5e^{\frac{1}{3}t}}{4+e^{\frac{1}{3}t}}$$

$$= \frac{5}{4e^{-\frac{1}{3}t} + 1}$$

8c. $e^{-\frac{1}{3}t} > 0$

$\therefore$ if $e^{-\frac{1}{3}t} = 0$ $\quad P = \frac{5}{1} = 5{,}000$

so $P < 5000$