Edexcel

A Level

A Level Maths

Edexcel Core Maths C3 January 2012 Model Solutions

Name:

M M E

Mathsmadeeasy.co.uk

Total Marks:

Edexcel Jan 12 - C3

1a. $\frac{d}{dx}\left[x^2 \ln(3x)\right]$ $= 2x\ln(3x) + \frac{3x^2}{3x}$

$= 2x\ln(3x) + x$

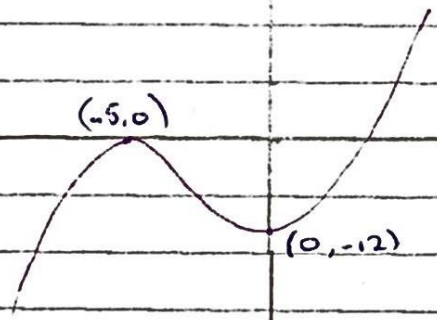
1b. $\frac{d}{dx}\left[\frac{\sin 4x}{x^3}\right]$ $f = \sin 4x$ $g = x^3$

$f' = 4\cos 4x$ $g' = 3x^2$

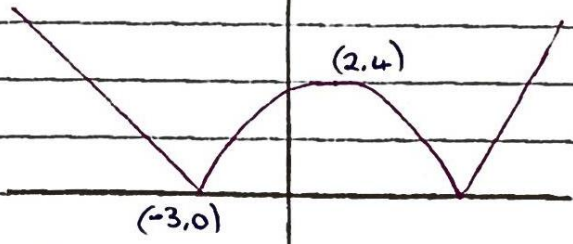
$= \frac{4x^3\cos 4x - 3x^2\sin 4x}{x^6}$

$= \frac{4x\cos 4x - 3\sin 4x}{x^4}$

2a.



2b.



3a $A = 20e^{1.5t}$, $t \geq 0$

when $t = 0$, $A = 20$

3b. $40 = 20e^{1.5t}$

$e^{1.5t} = 2$

$t = \frac{\ln 2}{1.5} = 0.4621\ldots$ hrs

$= 28$ mins

4. $x = 2\tan\left(y + \frac{\pi}{12}\right)$

$y = \frac{\pi}{4}$; $x = 2\tan\left(\frac{\pi}{4} + \frac{\pi}{12}\right) = 2\sqrt{3}$

$1 = 2\sec^2\left(y + \frac{\pi}{12}\right)\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{2}\cos^2\left(y + \frac{\pi}{12}\right)$

$y = \frac{\pi}{4}$; $\frac{dy}{dx} = \frac{1}{8}$ $\Rightarrow$ m of normal -8

$y - \frac{\pi}{4} = -8(x - 2\sqrt{3})$

5. $2\cot^2(3\theta) = 7\operatorname{cosec}(3\theta) - 5$

use: $\cot^2(3\theta) \equiv \operatorname{cosec}^2(3\theta) - 1$

$2\operatorname{cosec}^2(3\theta) - 7\operatorname{cosec}(3\theta) + 3 = 0$

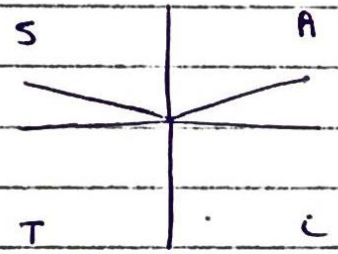
$(2\operatorname{cosec}(3\theta) - 1)(\operatorname{cosec}(3\theta) - 3) = 0$

$\sin(3\theta) = 2$ X

$\sin(3\theta) = 1/3$

$3\theta = 19.471$

$0 \leq 3\theta \leq 540°$



$3\theta = 19.471, 160.529$

$379.471, 520.529$

$\theta = 6.5°, 53.5°, 126.5°, 173.5°$

6a $f(x) = x^2 - 3x + 2\cos(x/2)$

$f(0.8) = 0.08212$

$f(0.9) = -0.0891$, change of sign, $\Rightarrow x \in (0.8, 0.9)$

6b. $f'(x) = 2x - 3 - \sin(x/2)$

$2x = 3 + \sin(x/2)$

$x = \dfrac{3 + \sin(x/2)}{2}$

6c. $x_{n+1} = \dfrac{3 + \sin(\frac{1}{2}x_n)}{2}$

$x_0 = 2$

$x_1 = 1.921$

$x_2 = 1.910$

$x_3 = 1.908$

6d. to round to 4 d.p. $x \in [1.90775, 1.90785)$

$f'(1.90775) = -0.0001634\ldots$

$f'(1.90785) = 0.0000766\ldots$

change of sign $\Rightarrow$ $1.90775 < x < 1.90785$

$\Rightarrow$ to 4 d.p. $x = 1.9078$

7a. $f: x \mapsto \frac{3(x+1)}{2x^2+7x-4} - \frac{1}{x+4}$, $x \in \mathbb{R}$, $x > \frac{1}{2}$

$$F(x) = \frac{3(x+1)-(2x-1)}{(2x-1)(x+4)} \cdot \frac{x+4}{(2x-1)(x+4)} = \frac{1}{2x-1}$$

7b. let $y = \frac{1}{2x-1}$

$$2xy - y = 1$$

$$x = \frac{1+y}{2y}$$

$$f^{-1}(x) = \frac{1+x}{2x}$$

7c. $x > 0$

7d. $g(x) = \ln(x+1)$

$$F(\ln(x+1)) = \frac{1}{7}$$

$$\frac{1}{2\ln(x+1)-1} = \frac{1}{7}$$

$$7 = 2\ln(x+1) - 1$$

$$8 = 2\ln(x+1)$$

$$x+1 = e^4$$

$$x = e^4 - 1$$

8a. $\dfrac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$ $\div \cos A\cos B$

$= \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

8b. $\tan(\theta + \pi/6) = \dfrac{\tan\theta + \tan(\pi/6)}{1 - \tan\theta\tan(\pi/6)}$

$= \dfrac{\tan\theta + 1/\sqrt{3}}{1 - \tan\theta \cdot 1/\sqrt{3}}$ $= \dfrac{\sqrt{3}\tan\theta + 1}{\sqrt{3} - \tan\theta}$

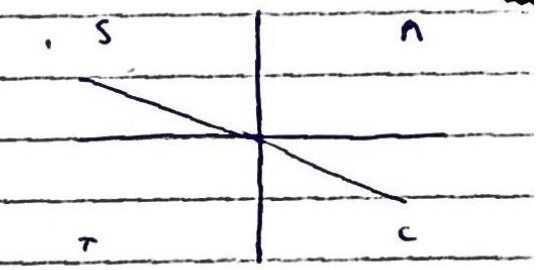
8c. $1 + \sqrt{3}\tan\theta = (\sqrt{3} - \tan\theta)\tan(\pi - \theta)$

$\tan(\theta + \pi/6)(\sqrt{3} - \tan\theta) = (\sqrt{3} - \tan\theta)\tan(\pi - \theta)$

$\tan(\theta + \pi/6) = \tan(\pi - \theta)$

$\theta + \pi/6 = \pi - \theta$ $\quad 0 \le \theta \le \pi$

$2\theta = 5\pi/6$ $\quad 0 \le 2\theta \le 2\pi$



$2\theta = 5\pi/6, \ 11\pi/6$

$\theta = 5\pi/12, \ 11\pi/12$