Edexcel

A Level

A Level Maths

Edexcel Core Maths C2 January 2012 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

Edexcel Jan 12 C2

1a. G.P. $a = 360, \quad r = 7/8$

$$u_{20} = ar^{19}$$
$$= 360.(7/8)^{19}$$
$$= 28.5 \quad (3sf)$$

1b.
$$S_{20} = \frac{a(1-r^{20})}{1-r}$$
$$= \frac{360(1-0.875^{20})}{1-7/8}$$
$$= 2680.68...$$
$$= 2680 \quad (3sf)$$

1c.
$$S_\infty = \frac{a}{1-r}$$
$$= \frac{360}{1-7/8}$$
$$= 2880$$

2. Radius is distance between $(0,0)$ and $(-1,7)$

$$r = \sqrt{(-1-0)^2+(7-0)^2}$$
$$= \sqrt{50}$$

$\therefore$ C $\quad (x+1)^2 + (y-7)^2 = 50$

3a.

$$\left(1+\frac{x}{4}\right)^8 = {}^8C_0 1^8 + {}^8C_1 1^7\left(\frac{x}{4}\right) + {}^8C_2 1^6\left(\frac{x}{4}\right)^2 + {}^8C_3 1^5\left(\frac{x}{4}\right)^3 + \dots$$

$$= 1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3 + \dots$$

3b.

$$\left(1+\frac{x}{4}\right)^8 = (1.025)^8 \quad ; \quad \frac{x}{4} = 0.025$$

$$x = 0.1$$

$$(1.025)^8 \approx 1 + 2(0.1) + \frac{7}{4}(0.1)^2 + \frac{7}{8}(0.1)^3 + \dots$$

$$\approx 1.218375\dots$$

$$= 1.22 \quad (3sf)$$

4a.

$$y = 3x^2$$

$$\log_3 y = \log_3(3x^2)$$

$$= \log_3 3 + \log_3 x^2$$

$$= \log_3 3 + 2\log_3 x$$

$$= 1 + 2\log_3 x$$

4b.

$$1 + 2\log_3 x = \log_3(28x - 9)$$

$$\log_3 y = \log_3(28x-9)$$

$$y = 28x - 9$$

$$3x^2 = 28x - 9 \quad (\text{Since } y = 3x^2)$$

$$3x^2 - 28x + 9 = 0$$

$$(3x-1)(x-9) = 0$$

$$x = 1/3 \text{ or } 9$$

5a. $f(x) = x^3 + ax^2 + bx + 3$

$f(-2) = 7$; $(-2)^3 + a(-2)^2 + b(-2) + 3 = 7$

$-8 + 4a - 2b + 3 = 7$

$4a - 2b = 12$

$2a - b = 6$ ①

5b. $f(1) = 4$; $(1)^3 + a(1)^2 + b(1) + 3 = 4$

$1 + a + b + 3 = 4$

$a + b = 0$ ②

'① + ②'

$3a = 6$

$a = 2$

$b = -2$

6a. $x = 2$, $y = 4$

$x = 2.5$, $y = 2.31$ (2dp)

6b. $h = 0.5$,

$R = \frac{1}{2}(0.5)\left\{(16.5 + 0) + 2(7.361 + 4 + 2.31 + 1.278 + 0.556)\right\}$

$= 11.8775$

$= 11.88$ (2dp)

6c. $\int_1^4 16x^{-2} - \frac{x}{2} + 1 \, dx$

$= \left[-16x^{-1} - \frac{1}{4}x^2 + x\right]_1^4$

$= \left(-16(4)^{-1} - \frac{1}{4}(4)^2 + 4\right) - \left(-16(1)^{-1} - \frac{1}{4}(1)^2 + 1\right)$

$= -4 - \left(-\frac{61}{4}\right)$

$= 45/4$

7a. $l = r\theta$; $6(0.95) = 5.7$

7b. $A = \frac{1}{2}r^2\theta$; $\frac{1}{2}(6)^2(0.95) = 17.1$

7c. $\hat{B} = 0.95$ (isosceles triangle)

$\hat{D} = \pi - 2(0.95)$ (angles in triangle sum to π)

$$\frac{AD}{\sin 0.95} = \frac{6}{\sin(\pi - 2(0.95))}$$

$$AD = \frac{6 \sin 0.95}{\sin(\pi - 2(0.95))}$$

$= 5.157447...$

$= 5.16$ (3sf)

7d. $BD = AD$ $(= 5.16)$

$BC = l$ $(= 5.7)$

$AB = AC$ (Since radii)

$\therefore$ $DC = 6 - AD$

$= 6 - 5.16$

$= 0.84$

$P = 5.16 + 5.7 + 0.84$

$= 11.7$

7e. A of Δ $= \frac{1}{2}ab\sin C$

$= \frac{1}{2}(6)(5.16)\sin(0.95)$

$= 12.5916...$

A of R = A of Sector − A of Δ

$= 17.1 - 12.5916...$

$= 4.5$ (2 s.f.)

8a. A of quarter circle : $\frac{1}{4}\pi x^2$

A of two rectangles : $2xy$

$\therefore$ $\frac{1}{4}\pi x^2 + 2xy = 4$

$\pi x^2 + 8xy = 16$

$16 - \pi x^2 = 8yx$

$y = \dfrac{16 - \pi x^2}{8x}$

8b. Circumference of full circle : $2\pi x$

$\therefore$ circumference of quarter circle : $\frac{1}{2}\pi x$

P of rectangles : $2x + 4y$

$\therefore$ $P = \frac{1}{2}\pi x + 2x + 4y$

$= \frac{1}{2}\pi x + 2x + \dfrac{4(16 - \pi x^2)}{8x}$

$= \frac{1}{2}\pi x + 2x + \dfrac{8}{x} - \frac{1}{2}\pi x$

$= \dfrac{8}{x} + 2x$

8c. $\dfrac{dP}{dx} = -8x^{-2} + 2$

for minimum set $\dfrac{dP}{dx} = 0$

$0 = -\dfrac{8}{x^2} + 2$

$\dfrac{8}{x^2} = 2$

$8 = 2x^2$

$x^2 = 4$

$x = \pm 2$

$\therefore$ $x = 2$ (can't have negative length)

8d. when $x = 2$, $y = \frac{16 - \pi(2)^2}{16}$

$= 1 - \frac{1}{4}\pi$

$= 21$ cm (nearest cm)

9i. $\sin(3x - 15) = 1/2$

$0 \leq x \leq 180°$

let $\phi = 3x - 15$

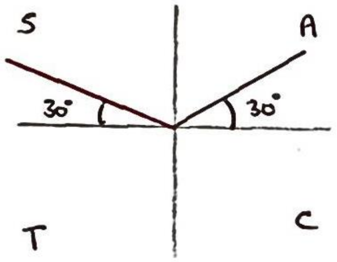
$0 \leq 3x \leq 540$

$\sin(\phi) = 1/2$

$-15 \leq 3x - 15 \leq 525$

P.V. $\phi = 30°$

$-15 \leq \phi \leq 525$



$\phi = 30°, 150°, 390°, 510°$

$3x - 15 = 30 \Rightarrow x = 15°$

$3x - 15 = 150 \Rightarrow x = 55°$

$3x - 15 = 390 \Rightarrow x = 135°$

$3x - 15 = 510 \Rightarrow x = 175°$

9ii. $y = \sin(ax - b)$

$\sin x \rightarrow \sin(ax)$ stretch x direction s.f. $\frac{1}{a}$

$\sin(ax) \rightarrow \sin(ax - b)$ translation $\frac{b}{a}$ units in x direction

Period of $y = \sin x = 2\pi$

Period of $y = \sin(ax - b) = PQ$

$= \frac{11}{10}\pi - \frac{1}{10}\pi$

$= \pi$

$\therefore$ stretch s.f. $\frac{1}{2} \Rightarrow \frac{1}{a} = \frac{1}{2}$

$a = 2$

Translation from O to P $= \pi/10$

$\therefore \frac{b}{a} = \frac{\pi}{10}$, $\frac{b}{2} = \frac{\pi}{10}$ $\therefore b = \pi/5$