

**Edexcel**

**A Level**

# **A Level Maths**

**Edexcel Core Maths C4 January  
2011 Model Solutions**

Name:



**Mathsmadeeasy.co.uk**

Total Marks:

Jan 11 Edexcel - C4

$$1. \int_0^{\pi/2} x \sin 2x \, dx$$

$$u = x$$

$$v' = \sin 2x$$

$$u' = 1$$

$$v = -\frac{1}{2} \cos 2x$$

$$\int_0^{\pi/2} x \sin 2x \, dx = -\frac{1}{2} \cos 2x + \frac{1}{2} \int \cos 2x \, dx$$

$$= \left[ -\frac{1}{2} \cos 2x + \frac{1}{4} \sin 2x \right]_0^{\pi/2}$$

$$= \pi/4$$

$$2. I = 16 - 16(0.5)^t$$

$$\text{use: } \frac{d}{dt} a^x = a^x \ln a$$

$$\frac{dI}{dt} = -16 \cdot 0.5^t \ln 0.5$$

$$\text{when } t = 3, \quad \frac{dI}{dt} = -2 \ln 0.5$$

$$= \ln 0.5^{-2}$$

$$= \ln 4$$

$$3a. \frac{5}{(x-1)(3x+2)} = \frac{A}{x-1} + \frac{B}{3x+2}$$

$$5 = A(3x+2) + B(x-1)$$

$$x=1 \Rightarrow 5 = 5A \Rightarrow A = 1$$

$$x = -2/3 \Rightarrow 5 = -5/3 B \Rightarrow B = -3$$

$$\text{so } \frac{5}{(x-1)(3x+2)} = \frac{1}{x-1} - \frac{3}{3x+2}$$

$$3b. \int \frac{1}{x-1} - \frac{3}{3x+2} dx = \ln(x-1) - \ln(3x+2) + c$$

$$3c. (x-1)(3x+2) \frac{dy}{dx} = 5y$$

$$\int \frac{1}{y} dy = \int \frac{5}{(x-1)(3x+2)} dx$$

$$\ln y = \ln(x-1) - \ln(3x+2) + c$$

$$\ln y = A \ln \left( \frac{x-1}{3x+2} \right)$$

$$\text{when } y=8, x=2 \Rightarrow \ln 8 = A \ln(1/8)$$

$$8 = \frac{A}{8} \Rightarrow A = 64$$

$$\text{so } y = \frac{64(x-1)}{3x+2}$$

$$4a. \quad A = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \quad B = \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$$

$$\vec{AB} = \begin{pmatrix} -3 \\ 5 \\ -3 \end{pmatrix}$$

$$4b. \quad r = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 5 \\ -3 \end{pmatrix}$$

$$4c. \quad C = \begin{pmatrix} 2 \\ p \\ -4 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} 1 \\ 3+p \\ -6 \end{pmatrix}$$

$$\perp \Rightarrow \vec{AC} \cdot \vec{AB} = 0$$

$$\begin{pmatrix} -3 \\ 5 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3+p \\ -6 \end{pmatrix} = -3 + 15 + 5p + 18 = 0$$

$$5p = -30$$

$$p = -6$$

$$4d. \quad |AC| = \sqrt{1^2 + (-3)^2 + (-6)^2}$$

$$= \sqrt{46}$$



$$5a. \quad (2-3x)^{-2} = [2(1-\frac{3}{2}x)]^{-2}$$

$$= 2^{-2} (1-\frac{3}{2}x)^{-2} = \frac{1}{4} \left( 1 - 2\left(-\frac{3}{2}x\right) + \frac{(-2)(-3)}{2!} \left(-\frac{3}{2}x\right)^2 + \frac{(-2)(-3)(-4)}{3!} \left(-\frac{3}{2}x\right)^3 + \dots \right)$$

$$= \frac{1}{4} \left( 1 + 3x + \frac{27}{4}x^2 + \frac{27}{2}x^3 + \dots \right)$$

$$= \frac{1}{4} + \frac{3}{4}x + \frac{27}{16}x^2 + \frac{27}{8}x^3 + \dots$$

$$5b. \quad (a+bx)(2-3x)^{-2} = (a+bx) \left( \frac{1}{4} + \frac{3}{4}x + \frac{27}{16}x^2 + \frac{27}{8}x^3 + \dots \right)$$

$$\frac{3}{4}a + \frac{1}{4}b = 0 \quad \Rightarrow \quad 3a + b = 0 \quad (1)$$

$$\frac{27}{16}a + \frac{3}{4}b = \frac{9}{16} \quad \Rightarrow \quad 27a + 12b = 9 \quad (2)$$

$$27a + 12b = 9 \quad \Rightarrow \quad b = 3, \quad A = -1$$

$$27a + 9b = 0$$

$$5c. \quad \frac{27}{8}a + \frac{27}{16}b = \frac{27(-1)}{8} + \frac{27(3)}{16}$$

$$= \frac{27}{16}$$

$$6a. \quad x = \ln t \qquad y = t^2 - 2$$

$$\frac{dx}{dt} = \frac{1}{t}$$

$$\frac{dy}{dt} = 2t$$

$$\frac{dy}{dx} = \frac{dt}{dx} \times \frac{dy}{dt} = t \times 2t = 2t^2$$

$$\text{when } t = 3, \quad x = \ln 3 \quad y = 7 \quad \frac{dy}{dx} = 2(3)^2 = 18$$

$$\Rightarrow \text{m of normal} = -\frac{1}{18}$$

$$\therefore y - 7 = -\frac{1}{18}(x - \ln 3)$$

$$6b. \quad x = \ln t \Rightarrow t = e^x \Rightarrow y = e^{2x} - 2$$

$$6c. \quad V = \pi \int_{\ln 2}^{\ln 4} (e^{2x} - 2)^2 dx = \pi \int_{\ln 2}^{\ln 4} e^{4x} - 4e^{2x} + 4 dx$$

$$= \pi \left[ \frac{1}{4} e^{4x} - 2e^{2x} + 4x \right]_{\ln 2}^{\ln 4}$$

$$= \pi \left( (64 - 32 + 4\ln 4) - (4 - 8 + 4\ln 2) \right)$$

$$= 36\pi + 4\pi \ln 2$$

$$= \pi (36 + 4\ln 2)$$

$$7a. \int_2^5 \frac{1}{4 + (x-1)^{1/2}} dx$$

$$x = 3$$

$$y = 0.1847$$

$$x = 5$$

$$y = 1/6 = 0.1667 \text{ to 4 dp.}$$

$$7b. h = \frac{5-2}{3} = 1$$

$$\int = \frac{1}{2} \left\{ (0.2 + 1/6) + 2(0.1847 + 0.1745) \right\}$$

$$= 0.543 \text{ to 3 dp.}$$

$$7c. x = (u-4)^2 + 1 \quad u = 4 + (x-1)^{1/2}$$

$$dx = 2(u-4) du$$

x	5	2
u	6	5

$$2 \int_5^6 \frac{(u-4)}{u} du$$

$$= 2 \left[ u - 4 \ln u \right]_5^6$$

$$= 12 - 8 \ln 6 - 10 + 8 \ln 5$$

$$= 10 + 8 \ln(5/6)$$