Edexcel

A Level

A Level Maths

Edexcel Core Maths C3 January 2011 Model Solutions

Name:

M M E
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Total Marks:

Edexcel Jan 11 C3

1a.

$$7\cos x - 24\sin x = R\cos(x+\alpha)$$
$$= R\cos\alpha\cos x - R\sin\alpha\sin x$$

$R = \sqrt{7^2 + 24^2} = 25$

$\cos x$; $7 = R\cos\alpha$

$\alpha = \cos^{-1}\left(\frac{7}{25}\right)$

$= 1.287$ (3d.p.)

1b.

$$7\cos x - 24\sin x = 25\cos(x + 1.287)$$

min. value of $\cos(x + 1.287) = -1$

$25 \times (-1) = -25$

1c.

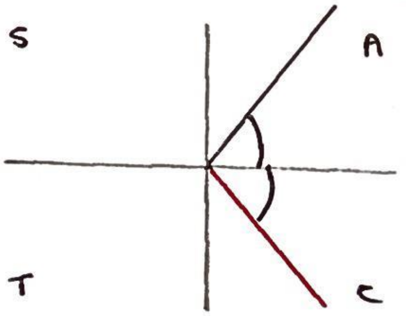
$25\cos(x + 1.287) = 10$ $\quad 0 \leq x < 2\pi$

$\cos(x + 1.287) = 0.4$

let $\phi = x + 1.287$ $\quad 1.287 < x < 7.570$

$\cos\phi = 0.4$

P.V. $\phi = 1.159$



$\phi = 5.124;\ 7.442$

$x + 1.287 = 5.124$ $\quad x = 3.84^c$

$x + 1.287 = 7.442$ $\quad x = 6.16^c$

2a.
$$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$$

$$\frac{(4x-1)(2x-1)}{2(x-1)(2x-1)} - \frac{3}{2(x-1)(2x-1)}$$

$$= \frac{8x^2-6x+1-3}{2(x-1)(2x-1)}$$

$$= \frac{8x^2-6x-2}{2(x-1)(2x-1)}$$

$$= \frac{4x^2-3x-1}{(x-1)(2x-1)}$$

$$= \frac{(4x+1)\cancel{(x-1)}}{\cancel{(x-1)}(2x-1)}$$

$$= \frac{4x+1}{2x-1}$$

2b.
$$f(x) = \frac{4x+1}{2x-1} - 2$$

$$= \frac{4x+1}{2x-1} - \frac{2(2x-1)}{2x-1}$$

$$= \frac{4x+1-4x+2}{2x-1}$$

$$= \frac{3}{2x-1}$$

2c.
$f = 3$ $\quad g = 2x-1$

$f' = 0$ $\quad g' = 2$

$$f'(x) = \frac{0(2x-1) - 3(2)}{(2x-1)^2}$$

$$= \frac{-6}{(2x-1)^2}$$

$$f'(2) = \frac{-6}{(2(2)-1)^2}$$

$$= -2/3$$

3.

$$2\cos 2\theta = 1 - 2\sin\theta \qquad 0^\circ \leq \theta < 360^\circ$$

use $\cos 2\theta \equiv 1 - 2\sin^2\theta$

$$2(1 - 2\sin^2\theta) = 1 - 2\sin\theta$$

$$2 - 4\sin^2\theta = 1 - 2\sin\theta$$

$$4\sin^2\theta - 2\sin\theta - 1 = 0$$

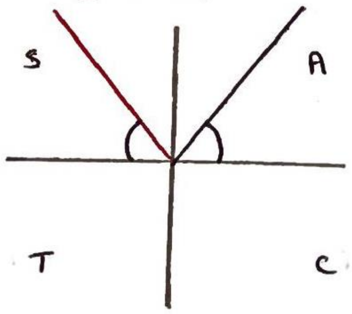
$$\sin\theta = \frac{2 \pm \sqrt{2^2 - 4(4)(-1)}}{2(4)}$$

$$= \frac{2 \pm 2\sqrt{5}}{8}$$

$$= \frac{1 \pm \sqrt{5}}{\cancel{8}4}$$

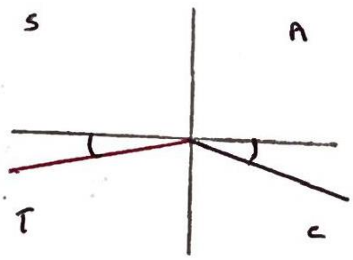
$$\sin\theta = \frac{1+\sqrt{5}}{\cancel{8}4} \quad \text{or} \quad \sin\theta = \frac{1-\sqrt{5}}{4}$$

P.V. $\theta = 54^\circ$



$\theta = 54^\circ, 126^\circ$

P.V. $= -18^\circ$



$\theta = 342^\circ, 198^\circ$

4a. when $t = 0$, $\theta = 90$

$$90 = 20 + Ae^{-k(0)}$$

$$90 = 20 + A$$

$$A = 70$$

4b. when $t = 5$, $\theta = 55$

$$55 = 20 + 70e^{-5k}$$

$$35 = 70e^{-5k}$$

$$\frac{1}{2} = e^{-5k}$$

$$\ln\left(\frac{1}{2}\right) = -5k$$

$$-\ln 2 = -5k \qquad \left(\ln\left(\tfrac{1}{2}\right) = \ln\left(2^{-1}\right) = -\ln 2\right)$$

$$k = \frac{1}{5}\ln 2$$

4c.

$$\theta = 20 + 70e^{-\frac{1}{5}\ln 2\, t}$$

$$\frac{d\theta}{dt} = 70\left(-\frac{1}{5}\ln 2\right)e^{-\frac{1}{5}\ln 2\, t}$$

when $t = 10$, $\frac{d\theta}{dt} = 70\left(-\frac{1}{5}\ln 2\right)e^{-\frac{1}{5}\ln 2 \times 10}$

$$= -2.426$$

$\therefore$ decreasing at 2.426° per min. (3dp)

5a. $f(x) = (8-x)\ln x \qquad x > 0$

$f(x) = 0$ when $8-x = 0 \Rightarrow x = 8$

or $\ln x = 0 \Rightarrow x = 1$

A $(1,0)$ B $(8,0)$

5b. $f(x) = (8-x)\ln x$

Product $f = (8-x)$ $g = \ln x$

$f' = -1$ $g' = \frac{1}{x}$

$f'(x) = -\ln x + \frac{8-x}{x}$

5c. Q is maximum $\Leftrightarrow$ at Q $f'(x) = 0$

$f'(3.5) = 0.03295\ldots$

$f'(3.6) = -0.0587\ldots$

change of sign $\Rightarrow$ $3.5 < Q < 3.6$

5d. at Q, $f'(x) = 0$

$$\frac{8-x}{x} - \ln x = 0$$

$$8 - x = x\ln x$$

$$8 = x\ln x + x$$

$$8 = x(\ln x + 1)$$

$$x = \frac{8}{\ln x + 1}$$

5e.

$$x_{n+1} = \frac{8}{1 + \ln x_n} \qquad x_0 = 3.55$$

$x_1 = 3.529$

$x_2 = 3.538$

$x_3 = 3.534$

6a.

$$f(x) = \frac{3-2x}{x-5} \qquad x \in \mathbb{R},\ x \neq 5$$

Let $y = \dfrac{3-2x}{x-5}$

$yx - 5y = 3 - 2x$

$yx + 2x = 3 + 5y$

$x(y+2) = 3 + 5y$

$x = \dfrac{3+5y}{y+2}$

$f^{-1}(x) = \dfrac{3+5x}{x+2}$

6b.

$-9 \leq g(x) \leq 4$

6c.

$gg(2)$ $\qquad g(2) = 0$

$g(0) = -6 = gg(2)$

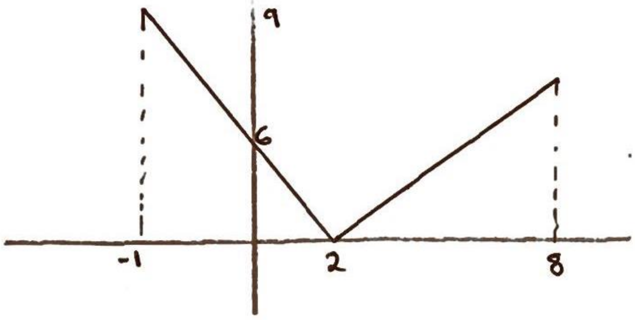
6d.

$fg(8)$ $\qquad g(8) = 4$

$$fg(8) = f(4) = \frac{3-2(4)}{4-5}$$

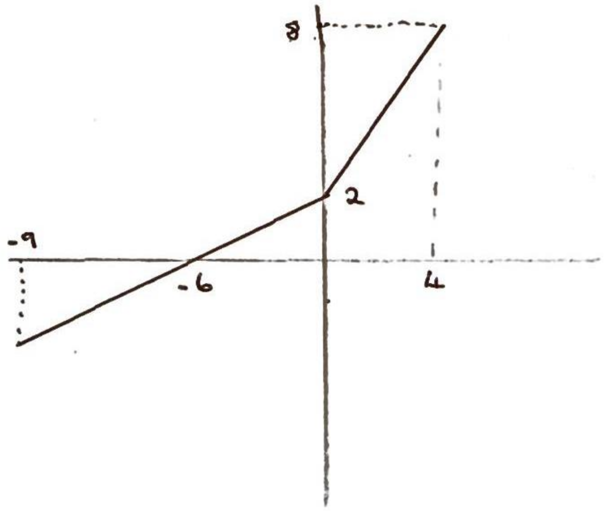
$= 5$

6ei.



$y = |g(x)|$

6eii.



$y = g^{-1}(x)$

(reflection in $y = x$)

6f.

domain of $g^{-1}(x)$ = range of $g(x)$

$\therefore \quad -9 \leq x \leq 4$

7a.

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}$$

$f = 3 + \sin 2x$

$f' = 2\cos 2x$

$g = 2 + \cos 2x$

$g' = -2\sin 2x$

$$\frac{dy}{dx} = \frac{2\cos 2x\,(2 + \cos 2x) - (3 + \sin 2x)(-2\sin 2x)}{(2 + \cos 2x)^2}$$

$$= \frac{4\cos 2x + 2\cos^2 2x + 6\sin 2x + 2\sin^2 2x}{(2 + \cos 2x)^2}$$

$$= \frac{4\cos 2x + 6\sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2 + \cos 2x)^2}$$

$$= \frac{6\sin 2x + 4\cos 2x + 2}{(2+\cos 2x)^2} \qquad (\cos^2\theta + \sin^2\theta \equiv 1 \quad \forall\, \theta \in \mathbb{R})$$

7b.

when $x = \pi/2$

$$y = \frac{3+\sin\pi}{2+\cos\pi} = 3 \qquad \therefore \quad C\ (\pi/2, 3)$$

when $x = \pi/2$,

$$\frac{dy}{dx} = \frac{6\sin\pi + 4\cos\pi + 2}{(2+\cos\pi)^2}$$

$$= -2$$

$$y - 3 = -2(x - \pi/2)$$

$$y - 3 = -2x + \pi$$

$$y = -2x + (3+\pi)$$

8a.

$$y = \sec x$$

$$= (\cos x)^{-1}$$

$$\frac{dy}{dx} = -1(-\sin x)(\cos x)^{-2}$$

$$= \frac{\sin x}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$= \tan x \sec x$$

8b. $x = \sec 2y$

$$\frac{dx}{dy} = 2\sec 2y \tan 2y$$

8c. $$\frac{dy}{dx} = \frac{1}{2\sec 2y \tan 2y}$$

$$= \frac{1}{2x \tan 2y} \qquad (x = \sec 2y)$$

$$1 + \tan^2 2y = \sec^2 2y$$

$$\tan^2 2y = \sec^2 2y - 1$$

$$\tan 2y = \sqrt{\sec^2 2y - 1}$$

$$= \sqrt{x^2 - 1}$$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{2x\sqrt{x^2-1}}$$