Edexcel

A Level

A Level Maths

Edexcel Core Maths C4 January 2010 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

Edexcel Jan 10 C4

1a. $\sqrt{1-8x}$

$$(1-8x)^{1/2} = 1 + \left(\tfrac{1}{2}\right)(-8x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(-8x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-8x)^3 + \ldots$$

$$= 1 - 4x - 8x^2 - 32x^3 + \ldots$$

1b. when $x = \frac{1}{100}$; $\sqrt{1-8\left(\frac{1}{100}\right)}$

$$= \sqrt{\frac{92}{100}}$$

$$= \sqrt{\frac{23}{25}}.$$

$$= \frac{\sqrt{23}}{5}$$

1c.

$$\frac{\sqrt{23}}{5} \approx 1 - 4\left(\frac{1}{100}\right) - 8\left(\frac{1}{100}\right)^2 - 32\left(\frac{1}{100}\right)^3$$

$$\approx 0.959168$$

$$\sqrt{23} \approx 5(0.959168)$$

$$\approx 4.79584$$

2a.

$x = 2$, $y = 1.386$

$x = 2.5$, $y = 2.291$

2b. $h = 0.5$

$$R \approx \tfrac{1}{2}(0.5)\left\{(0 + 5.545) + 2(0.608 + 1.386 + 2.291 + 3.296 + 4.385)\right\}$$

$$\approx 7.36925$$

$$= 7.37 \quad (2dp)$$

2ci.

$$\int x \ln x \, dx \qquad \text{Parts:} \quad u = \ln x \qquad v' = x$$
$$u' = \frac{1}{x} \qquad v = \frac{1}{2}x^2$$

$$\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}\frac{x^2}{x} \, dx$$
$$= \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x \, dx$$
$$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$$

2cii.

$$\int_1^4 x \ln x \, dx = \left[\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2\right]_1^4$$
$$= \left(\frac{1}{2}(4)^2 \ln 4 - \frac{1}{4}(4)^2\right) - \left(\frac{1}{2}1^2 \ln 1 - \frac{1}{4}(1)^2\right)$$
$$= (8\ln 4 - 4) - (-1/4)$$
$$= (16\ln 2 - 4) - (-1/4)$$
$$= 16\ln 2 - 15/4$$
$$= \frac{1}{4}(64\ln 2 - 15)$$

3a.

$$\cos 2x + \cos 3y = 1, \quad -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}, \quad 0 \leq y \leq \frac{\pi}{6}$$
$$-2\sin 2x - 3\sin 3y \frac{dy}{dx} = 0$$
$$3\sin 3y \frac{dy}{dx} = -2\sin 2x$$
$$\frac{dy}{dx} = -\frac{2\sin 2x}{3\sin 3y}$$

3b. when $x = \frac{\pi}{6}$,

$$\cos\left(2\pi/6\right) + \cos 3y = 1$$

$$\cos 3y = 1/2$$

$$3y = \pi/3$$

$$y = \pi/9 \quad \Rightarrow \quad P\left(\pi/6, \pi/9\right)$$

3c. At P, $\frac{dy}{dx} = -\frac{2\sin\left(2\pi/6\right)}{3\sin\left(3\pi/9\right)}$

$$= -\frac{2}{3}$$

$\therefore$ m of tangent $= -2/3$

$$y - \frac{\pi}{9} = -\frac{2}{3}\left(x - \pi/6\right) \qquad \times 3$$

$$3y - \frac{\pi}{3} = -2x + \frac{\pi}{3} \qquad \times 3$$

$$9y - \pi = -6x + \pi$$

$$6x + 9y - 2\pi = 0$$

4a. l_1 $\quad \underline{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$

l_2 $\quad \underline{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$

A at $\begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix}$

4b. $\cos\theta = \frac{a \cdot b}{|a|\,|b|}$

$$\cos\theta = \frac{4(3) + (-1)(-4) + 3(1)}{\sqrt{4^2+1^2+3^2}\ \sqrt{3^2+4^2+1^2}}$$

$$= \frac{19}{26}$$

4c.

when $\lambda = 4$

$$l_1 : \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + 4\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -6 + 4(4) \\ 4 + 4(-1) \\ -1 + 4(3) \end{pmatrix}$$

$$= \begin{pmatrix} 10 \\ 0 \\ 11 \end{pmatrix}$$

4d.

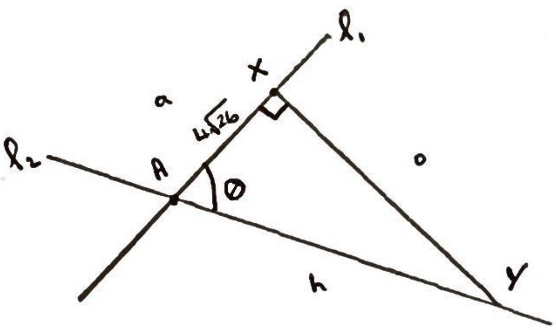
$A\ (-6, 4, -1) \qquad X\ (10, 0, 11)$

$$\overrightarrow{AX} = \begin{pmatrix} 16 \\ -4 \\ 12 \end{pmatrix}$$

4e.

$$|\overrightarrow{AX}| = \sqrt{16^2+4^2+12^2} = \sqrt{416} = 4\sqrt{26}$$

4f.



$$\cos\theta = \frac{AX}{AY}$$

$$\cos\theta = \frac{4\sqrt{26}}{AY} \qquad \cos\theta = \left(\frac{19}{26}\right)$$

$$AY = \frac{4\sqrt{26}}{19/26}$$

$$= 27.9 \quad (3sf)$$

5a. $\int \frac{9x+6}{x}\,dx$

$= \int 9 + \frac{6}{x}\,dx$

$= 9x + 6\ln x + c$

5b. $\frac{dy}{dx} = \frac{9x+6}{x}\,y^{1/3}$

$\int y^{-1/3}\,dy = \int \frac{9x+6}{x}\,dx$

$\frac{3}{2}y^{2/3} = 9x + 6\ln x + c$

when $y = 8, \quad x = 1$

$\frac{3}{2}(8)^{2/3} = 9(1) + 6\ln 1 + c$

$6 = 9 + c \quad \Rightarrow \quad c = -3$

$\frac{3}{2}y^{2/3} = 9x + 6\ln x - 3 \qquad \times \frac{2}{3}$

$y^{2/3} = 6x + 4\ln x - 2$

$y^2 = (6x + 4\ln x - 2)^3$

6. $\frac{dA}{dt} = 1.5$

we want $\frac{dr}{dt}$

$\frac{dA}{dt} \times ? = \frac{dr}{dt}$

$\frac{dA}{dt} \times \frac{dr}{dA} = \frac{dr}{dt}$

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r \quad \Rightarrow \quad \frac{dr}{dA} = \frac{1}{2\pi r}$$

$$\frac{dr}{dt} = \frac{dA}{dt} \times \frac{dr}{dA}$$

$$= 1.5 \times \frac{1}{2\pi r}$$

$$= \frac{3}{4\pi r}$$

when $A = 2$; $\quad 2 = \pi r^2$

$$r = \sqrt{\frac{2}{\pi}}$$

$$\frac{dr}{dt} = \frac{3}{4\pi\left(\sqrt{\frac{2}{\pi}}\right)} = 0.299 \quad (3sf)$$

7a. $x = 5t^2 - 4 \qquad y = t(9 - t^2)$

at A and B , $y = 0$

$$0 = t(9 - t^2)$$

$$= t(3 - t)(3 + t)$$

$\therefore \quad t = 0, \ 3, \text{ or } -3$

when $t = 0$, $\quad x = 5(0)^2 - 4$

$$= -4 \quad \Rightarrow \quad A\ (-4, 0)$$

when $t = \pm 3$, $\quad x = 5(\pm 3)^2 - 4$

$$= 41 \quad \Rightarrow \quad B\ (41, 0)$$

7b.

$$R = 2\int_{-4}^{41} y \quad dx$$

$$x = 5t^2 - 4$$

$$\frac{dx}{dt} = 10t$$

x	41	-4
t	3	0

$$\left(\begin{array}{ll} 5t^2 - 4 = 41 & \Rightarrow t = 3 \\ 5t^2 - 4 = -4 & \Rightarrow t = 0 \end{array} \right)$$

$$R = 2\int_0^3 t(9-t^2)\ 10t \quad dt$$

$$R = 20\int_0^3 9t^2 - t^4 \quad dt$$

$$= 20\left[3t^3 - \frac{1}{5}t^5\right]_0^3$$

$$= 20\left(\left(3(3)^3 - \frac{1}{5}(3)^5\right) - (0)\right)$$

$$= 648$$

8a.

$$\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{4-x^2}} \quad dx$$

$$x = 2\cos u$$

$$dx = -2\sin u \quad du$$

$$x^2 = 4\cos^2 u$$

$$\sqrt{4-x^2} = \sqrt{4-4\cos^2 u}$$

$$= 2\sqrt{1-\cos^2 u}$$

$$= 2\sqrt{\sin^2 u}$$

$$= 2\sin u$$

x	$\sqrt{2}$	1
u	$\pi/4$	$\pi/3$

$$\left(\begin{array}{ll} \sqrt{2} = 2\cos u & \Rightarrow u = \pi/4 \\ 1 = 2\cos u & \Rightarrow u = \pi/3 \end{array} \right)$$

$$= \int_{\pi/3}^{\pi/4} \frac{1}{4\cos^2 u \cdot 2\sin u} \cdot -2\sin u \quad du$$

$$= -\frac{1}{4}\int_{\pi/3}^{\pi/4} \sec^2 u \quad du$$

$$= -\frac{1}{4}\left[\tan u\right]_{\pi/3}^{\pi/4}$$

$$= -\frac{1}{4}\left(1 - \sqrt{3}\right)$$

$$= \frac{\sqrt{3} - 1}{4}$$

8b.

$$V = \pi \int_1^{\sqrt{2}} y^2 \, dx$$

$$= \pi \int_1^{\sqrt{2}} \frac{16}{x^2\sqrt{4-x^2}} \, dx$$

$$= 16\pi \int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{4-x^2}} \, dx$$

$$= 16\pi \left(\frac{\sqrt{3}-1}{4} \right)$$

$$= 4\pi(\sqrt{3}-1)$$