Edexcel

A Level

A Level Maths

Edexcel Core Maths C3 January 2010 Model Solutions

Name:

MME

Mathsmadeeasy.co.uk

Total Marks:

Edexcel Jan 10 C3

1.

$$\frac{x+1}{3x^2-3} - \frac{1}{3x+1}$$

$$\frac{x+1}{3(x^2-1)} - \frac{1}{3x+1}$$

$$\frac{x+1}{3(x+1)(x-1)} - \frac{1}{3x+1}$$

$$\frac{1}{3(x-1)} - \frac{1}{3x+1}$$

$$\frac{3x+1 - 3(x-1)}{3(x-1)(3x+1)}$$

$$\frac{4}{3(x-1)(3x+1)}$$

2a.

$$f(x) = x^3 + 2x^2 - 3x - 11$$

$$x^3 + 2x^2 - 3x - 11 = 0$$

$$x^3 + 2x^2 = 3x + 11$$

$$x^2(x+2) = 3x + 11$$

$$x^2 = \frac{3x+11}{x+2}$$

$$x = \sqrt{\frac{3x+11}{x+2}}$$

2b. $x_{n+1} = \sqrt{\dfrac{3x_n + 11}{x_n + 2}}$

$x_1 = 0$

$x_2 = 2.345$

$x_3 = 2.0397$

$x_4 = 2.059$

2c. if $\alpha = 2.057$ to 3dp then $2.0565 < \alpha < 2.0575$

$f(2.0565) = -0.01378\ldots$

$f(2.0575) = 0.004140\ldots$

change of sign $\therefore$ $2.0565 < \alpha < 2.0575$

$\therefore$ $\alpha = 2.057$ to 3dp

3a. $5\cos x - 3\sin x = R\cos(x + \alpha)$

$= R\cos x\cos\alpha - R\sin x\sin\alpha$

$R = \sqrt{5^2 + 3^2}$

$= \sqrt{34}$

$\cos x$: $5 = R\cos\alpha$

$\alpha = \cos^{-1}\left(\dfrac{5}{\sqrt{34}}\right)$

$= 0.5404\ldots$

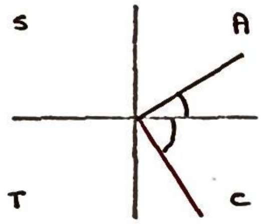
$5\cos x - 3\sin x = \sqrt{34}\cos(x + 0.5404)$

3b. $\cos(x + 0.5404) = \dfrac{4}{\sqrt{34}}$ $\quad 0 \leq x < 2\pi$

let $\phi = x + 0.5404$

$\cos\phi = \frac{4}{\sqrt{34}}$ $\qquad 0.5404 \leq \phi < 6.8236$

P.V. $\phi = 0.8148$



$\phi = 0.8148^c, \; 5.4684^c$

$x = \phi - 0.5404^c$

$x = 0.27^c, \; 4.93^c \quad (2dp)$

4i. $y = \frac{\ln(x^2+1)}{x}$

Quotient Rule

$f = \ln(x^2+1)$ $\qquad g = x$

$f' = \frac{2x}{x^2+1}$ $\qquad g' = 1$

$$\frac{dy}{dx} = \frac{x \cdot \frac{2x}{x^2+1} - 1\ln(x^2+1)}{x^2}$$

$$= \frac{\frac{2x^2}{x^2+1} - \ln(x^2+1)}{x^2}$$

$$= \frac{2}{x^2+1} - \frac{1}{x^2}\ln(x^2+1)$$

4ii. $x = \tan y$

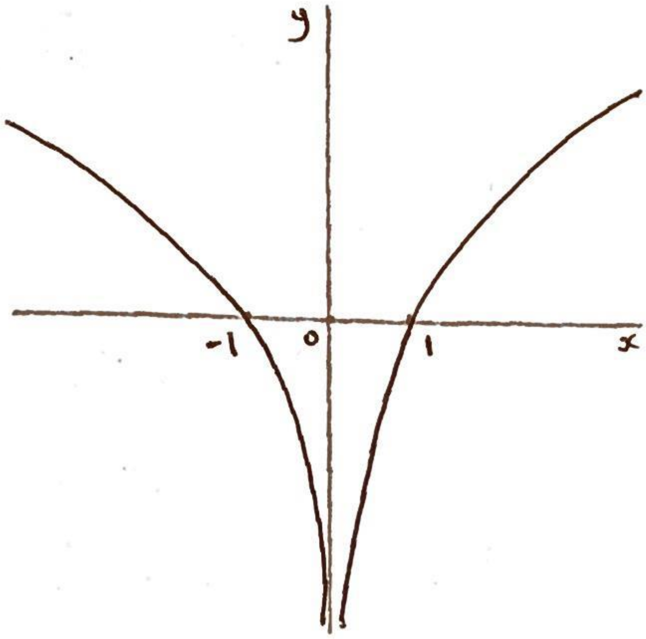
$\frac{dx}{dy} = \sec^2 y$ $\qquad (\sec^2\theta \equiv 1 + \tan^2\theta \;\; \forall\, \theta \in \mathbb{R})$

$\frac{dx}{dy} = 1 + \tan^2 y$ $\qquad (x = \tan y)$

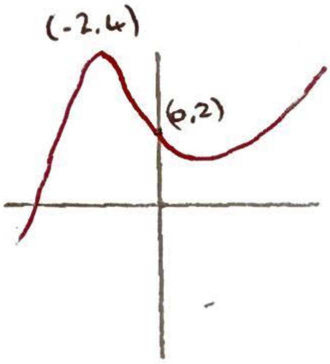
$\frac{dx}{dy} = 1 + x^2$

$\frac{dy}{dx} = \frac{1}{1+x^2}$

5.

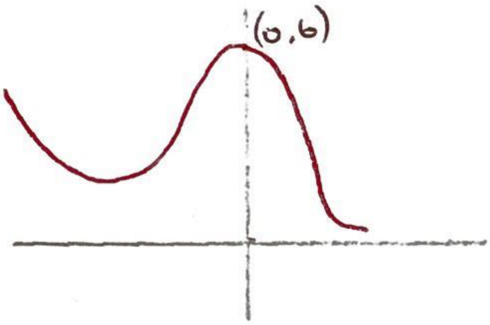


6i.

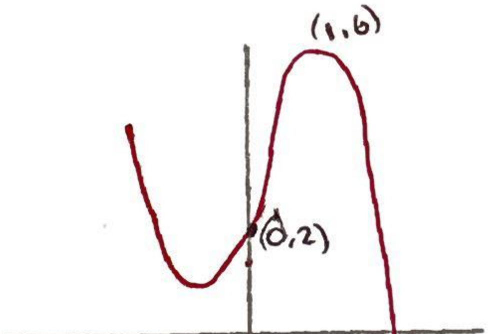


$y = f(-x) + 1$

6ii.



$y = f(x+2) + 3$



$y = 2f(2x)$

7a.

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left((\cos x)^{-1}\right)$$

$$= -1(-\sin x)(\cos x)^{-2}$$

$$= \frac{\sin x}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$= \sec x \tan x$$

7b.

$$y = e^{2x} \sec 3x$$

$$\frac{dy}{dx} = 2e^{2x}\sec 3x + 3e^{2x}\sec 3x \tan 3x$$

7c.

at minimum $\frac{dy}{dx} = 0$

$$2e^{2x}\sec 3x + 3e^{2x}\sec 3x \tan 3x = 0 \qquad -\frac{\pi}{6} < x < \pi/6$$

$e^{2x} > 0 \quad \forall\, x \in \mathbb{R}$ - so can divide by e^{2x}

$$2\sec 3x + 3\sec 3x \tan 3x = 0$$

$\sec 3x \neq 0, \; \forall\, x : -\pi/6 < x < \pi/6$ - so can divide by $\sec 3x$

$$2 + 3\tan 3x = 0$$

$$\tan 3x = -2/3$$

$$3x = \tan^{-1}(-2/3)$$

$$3x = -0.58800$$

$$x = -0.19600$$

$$y = e^{2(-0.19600)}\sec(3(-0.19600))$$

$$= 0.812$$

$\therefore$ minimum at

$(-0.196, 0.812)$

8. $\operatorname{cosec}^2 2x - \cot 2x = 1 \qquad 0 \leq x \leq 180$

use $\operatorname{cosec}^2\theta \equiv 1 + \cot^2\theta$

$1 + \cot^2 2x - \cot 2x = 1$

$\cot 2x(\cot 2x - 1) = 0$

$\cot 2x = 0$

let $\phi = 2x$

$\cot\phi = 0 \qquad 0 \leq \phi \leq 360$

$\tan\phi = \infty$

P.V. $\phi = 90°$ (since $y = \tan x$ asymptotic at $x = 90$)

$\phi = 90°, 270°$

$2x = \phi$

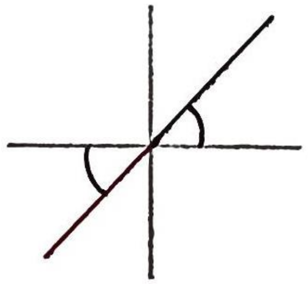
$x = 45°, 135°$

$\cot 2x = 1$

let $\phi = 2x \qquad 0 \leq \phi \leq 360$

$\tan\phi = 1$

P.V. $\phi = 45°$



$\phi = 45°, 225°$

$2x = \phi$

$x = 22.5°, 112.5°$

9.i.a $\ln(3x-7) = 5$

$3x - 7 = e^5$

$3x = e^5 + 7$

$x = \frac{1}{3}(e^5 + 7)$

9.i.b $3^x e^{7x+2} = 15$

$\ln\left(3^x e^{7x+2}\right) = \ln 15$

$\ln 3^x + \ln e^{7x+2} = \ln 15$

$x \ln 3 + 7x + 2 = \ln 15$

$x(\ln 3 + 7) = \ln 15 - 2$

$$x = \frac{\ln 15 - 2}{\ln 3 + 7}$$

9.ii.a. $f(x) = e^{2x} + 3 \quad x \in \mathbb{R}$

$g(x) = \ln(x-1) \quad x \in \mathbb{R},\ x > 1$

let $y = e^{2x} + 3$

$y - 3 = e^{2x}$

$\ln(y-3) = 2x$

$x = \frac{1}{2}\ln(y-3)$

$\therefore \quad f^{-1}(x) = \frac{1}{2}\ln(x-3)$

9.ii.b. $fg(x) = e^{2\ln(x-1)} + 3$

$= e^{\ln(x-1)^2} + 3$

$= (x-1)^2 + 3$

$fg(x) \geq 3$