Edexcel

A Level

A Level Maths

Edexcel Core Maths C2 January 2010 Model Solutions

Name:

M M E

Mathsmadeeasy.co.uk

Total Marks:

Edexcel Jan 2010 C2

1. $(3-x)^6 = {}^6C_0 \cdot 3^6 + {}^6C_1 3^5(-x) + {}^6C_2 3^4(-x)^2 + \ldots$

$= 729 - 1458x + 1215x^2$

2a. $5\sin x = 1 + 2\cos^2 x$

$5\sin x = 1 + 2(1 - \sin^2 x)$

$5\sin x = 1 + 2 - 2\sin^2 x$

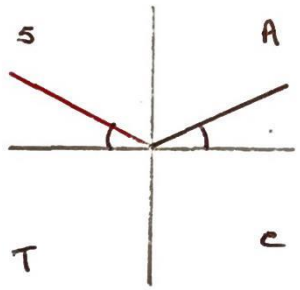
$2\sin^2 x + 5\sin x - 3 = 0$

2b. $(2\sin x - 1)(\sin x + 3) = 0$ $\quad 0 \leq x < 360°$

$2\sin x - 1 = 0$ or $\sin x + 3 = 0$

$\sin x = 1/2$ $\quad \sin x = -3$ - no solutions

P.V. $x = 30°$



$\therefore \quad x = 30°, 150°$

3a. $f(x) = 2x^3 + ax^2 + bx - 6$

$f(x)$ divided by $(2x-1)$ has remainder -5 $\quad \therefore \quad f(1/2) = -5$

$f(1/2) = 2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + b(1/2) - 6$

$-5 = \frac{1}{4} + \frac{1}{4}a + \frac{1}{2}b - 6$

$\frac{3}{4} = \frac{1}{4}a + \frac{1}{2}b \quad \times 4$

$3 = a + 2b \quad (1)$

$f(-2) = 0$

$f(-2) = 2(-2)^3 + a(-2)^2 + b(-2) - 6$

$0 = -16 + 4a - 2b - 6$

$22 = 4a - 2b$ ②

'① + ②' $25 = 5a$ $a = 5$

$b = -1$

3b.

$$x+2 \overline{)\,2x^3 + 5x^2 - x - 6}$$

$2x^2 + x - 3$

$2x^3 + 4x^2$ ↓ ↓

$x^2 - x$

$x^2 + 2x$

$-3x - 6$

$-3x - 6$

0

$\therefore \quad f(x) = (x+2)(2x^2 + x - 3)$

$= (x+2)(2x+3)(x-1)$

4a.

$$\frac{\sin \hat{C}}{5} = \frac{\sin 0.6}{4}$$

$$C = \sin^{-1}\left(\frac{5 \sin 0.6}{4}\right)$$

$= 0.783556\ldots$

angles in Δ sum to π

$\hat{A} + \hat{B} + \hat{C} = \pi$

$0.6 + \hat{B} + 0.783556\ldots = \pi$

$\hat{B} = \pi - 0.6 - 0.783556\ldots$

$= 1.75803\ldots$

$= 1.76 \quad (3sf)$

4b. A of Sector $= \frac{1}{2}r^2\theta$

$\theta = \pi - 1.76$ (angles on a straight line)

$= 1.381\ldots$

$A = \frac{1}{2}(4)^2(1.381\ldots)$

$= 11.05274123$

A of $\Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(5)(4)\sin(1.76)$

$= 9.821543171$

Total area $= 20.87428\ldots$

$= 20.9$ (3sf)

5a. $\log_x 64 = 2 \iff x^2 = 64$

$x = 8$ (since $x > 0$)

5b. $\log_2(11-6x) = 2\log_2(x-1) + 3$

$\log_2(11-6x) = \log_2(x-1)^2 + 3$

$\log_2(11-6x) = \log_2(x-1)^2 + \log_2 8$ $\quad 3 = \log_2 8$ since $2^3 = 8$

$\log_2(11-6x) = \log_2[8(x-1)^2]$

$11-6x = 8(x-1)^2$

$11-6x = 8(x^2-2x+1)$

$8x^2-16x+8 = 11-6x$

$8x^2+10x-3 = 0$

$(4x+1)(2x-3) = 0$

$x = -1/4$ or $3/2$

6a. $18{,}000 \times 0.8^3 = 9216$

6b. G.P. $a = 18{,}000$ $r = 0.8$

$ar^{n-1} < 1000$

$18000\,(0.8)^{n-1} < 100$

$0.8^{n-1} < 1/18$

$(n-1)\log 0.8 < \log(1/18)$

$n-1 > \dfrac{\log(1/18)}{\log 0.8}$ inequality flips since $\log 0.8 < 0$

$n > 12.95\ldots$

$n = 13$

6c. G.P. $a = 200$ $r = 1.12$

1st	2nd	3rd	4th	5th
200	224	250.88	280.9856	314.703...

So £314.70 in 5th year

6d. $S_{15} = \dfrac{a(r^{15}-1)}{r-1}$

$= \dfrac{200\,(1.12^{15}-1)}{1.12-1}$

$= 7455.9429\ldots$

$= £7455.94$

7a. $y = x^2 - 5x + 4$ at L and M $y = 0$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 1$ and $x = 4$

$\therefore$ L(1,0) M(4,0)

7b. $x = 5$ when $y = 4$

$4 = 5^2 - 5(5) + 4$

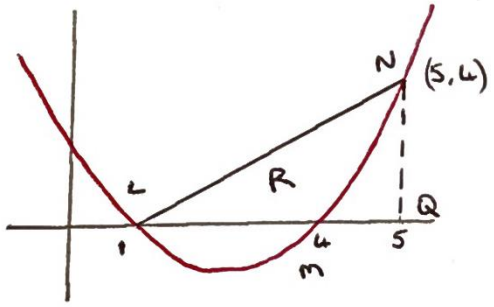
$= 25 - 25 + 4$

$= 4$ $\therefore$ N lies on C

7c. $\int x^2 - 5x + 4 \, dx$

$= \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x + C$

7d.



A of Δ LQN

$= \frac{1}{2}(4 \times 4) = 8$

A under curve $= \int_4^5 x^2 - 5x + 4 \, dx$

$= \left(\frac{1}{3}(5)^3 - \frac{5}{2}(5)^2 + 4(5)\right) - \left(\frac{1}{3}(4)^3 - \frac{5}{2}(4)^2 + 4(4)\right)$

$= \frac{11}{6}$

$\therefore$ R $= 8 - \frac{11}{6}$

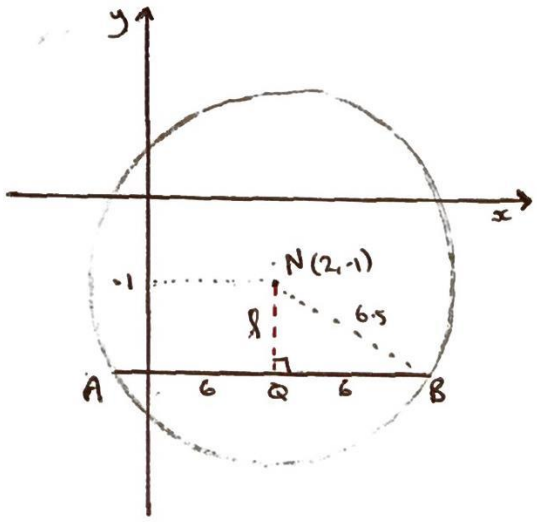
$= 37/6$

8a. $(x-2)^2 + (y+1)^2 = \frac{169}{4}$

Centre $(2, -1)$

8b. radius $= \sqrt{\frac{169}{4}} = \frac{13}{2}$

8c.



so x coord. of A : $2-6 = -4$

x coord. of B : $2+6 = 8$

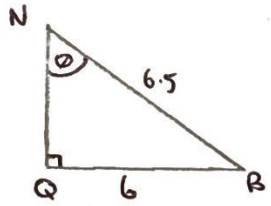
NB : 6.5 (radius) $\quad 6.5^2 = \ell^2 + 6^2$

$\ell = 2.5$

$\therefore$ y coords of A and B $-1 - 2.5 = -3.5$

A $(-4, -3.5)$ B $(8, -3.5)$

8d.



$\theta = \frac{1}{2} A\hat{N}B$

$\sin\theta = \frac{6}{6.5}$

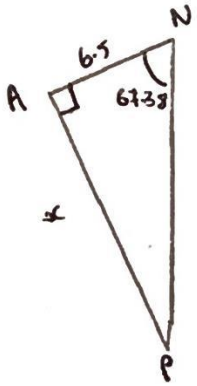
$\theta = 67.38\ldots$

$\therefore A\hat{N}B = 2 \times 67.38\ldots$

$= 134.76\ldots$

$= 134.8°$

8e.



$\tan(67.38) = \frac{x}{6.5}$

$x = 6.5 \tan(67.38)$

$= 15.599\ldots$

$= 15.6$ (3sf)

9a. $y = 12x^{1/2} - x^{3/2} - 10$

$\frac{dy}{dx} = 6x^{-1/2} - \frac{3}{2}x^{1/2}$

at st. pt. $\frac{dy}{dx} = 0$; $\quad 0 = \frac{6}{\sqrt{x}} - \frac{3}{2}\sqrt{x} \qquad (\times \sqrt{x})$

$0 = 6 - \frac{3}{2}x$

$\frac{3}{2}x = 6$

$x = 4$

when $x = 4$; $\quad y = 12(4)^{1/2} - (4)^{3/2} - 10$

$= 6$

$(4, 6)$

9b $\frac{d^2y}{dx^2} = -3x^{-3/2} - \frac{3}{4}x^{-1/2}$

9c. when $x = 4$, $\quad \frac{d^2y}{dx^2} = -3(4)^{-3/2} - \frac{3}{4}(4)^{-1/2}$

$= -3/4$

$\frac{d^2y}{dx^2} < 0 \quad \therefore$ maximum