AQA

A Level

A Level Maths

AQA Core Maths C1 June 2014 Model Solutions

Name:

M M E

Mathsmadeeasy.co.uk

Total Marks:

Jun 14 C1 - AQA

1ai. $A = (-1, 2)$ $B = (3, -5)$

$$\text{grad} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2 - -5}{-1 - 3} = -\frac{7}{4}$$

1aii. $y = -\frac{7}{4}x + c$

$$2 = \frac{7}{4}x + c$$

$$8 = 7 + 4c \Rightarrow c = 1/4$$

$$y = -\frac{7}{4}x + \frac{1}{4}$$

$$7x + 4y = 1$$

1bi. $m = \left(\frac{-1+3}{2}, \frac{2+-5}{2}\right) = (1, -3/2)$

1bii. $\perp \Rightarrow m = \frac{4}{7}$

$$y = \frac{4}{7}x + c$$

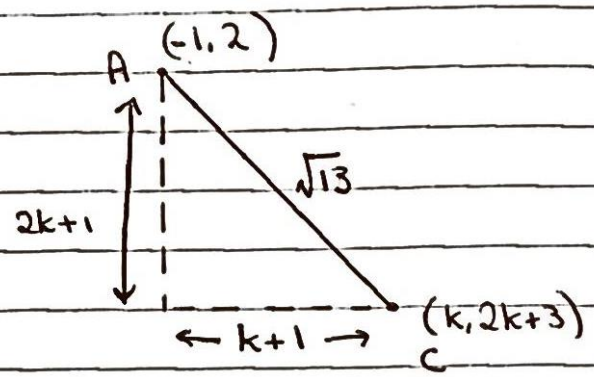
$$-\frac{3}{2} = \frac{4}{7} + c$$

$$c = -\frac{29}{14}$$

$$y = \frac{4}{7}x - \frac{29}{14}$$

$$14y = 8x - 29$$

1c.



$$\Rightarrow (2k+1)^2 + (k+1)^2 = 13$$

$$4k^2 + 4k + 1 + k^2 + 2k + 1 = 13$$

$$5k^2 + 6k - 11 = 0$$

$$(5k+11)(k-1) = 0$$

$$\Rightarrow k = -11/5 \text{ or } 1$$

2. length : $(9+5\sqrt{3})$ area : $(15+7\sqrt{3})$

width : a/l

$$\Leftrightarrow \frac{(15+7\sqrt{3})(9-5\sqrt{3})}{(9+5\sqrt{3})(9-5\sqrt{3})}$$

$$= \frac{135 - 75\sqrt{3} + 63\sqrt{3} - 105}{9^2 - (5\sqrt{3})^2}$$

$$= \frac{30 - 12\sqrt{3}}{81 - 75}$$

$$= \frac{30 - 12\sqrt{3}}{6} = 5 - 2\sqrt{3}$$

3ai. $y = 2x^5 + 5x^4 - 1$

$\frac{dy}{dx} = 10x^4 + 20x^3$

3aii. $\frac{d^2y}{dx^2} = 40x^3 + 60x^2$

3bi. at P, $x = -1$, $\frac{dy}{dx} = 10(-1)^4 + 20(-1)^3$

$= 10 - 20$

$= -10$

$\frac{dy}{dx} < 0 \Rightarrow$ decreasing function

3bi. y coord of P: $y = 2(-1)^5 + 5(-1)^4 - 1$

$= -2 + 5 - 1$

$= 2$

m of tangent is -10

$y = -10x + c$

$2 = 10 + c \Rightarrow c = -8$

$y = -10x - 8$

3c. $Q = (-2, 15)$

at Q, $\frac{dy}{dx} = 10(-2)^4 + 20(-2)^3$

$= 160 - 160$

$= 0 \Rightarrow$ stat. point

at Q, $\frac{d^2y}{dx^2} = 40(-2)^3 + 60(-2)^2$

$= -320 + 240$

$= -80$, $\frac{d^2y}{dx^2} < 0 \Rightarrow$ maximum

4ai. $16 - 6x - x^2 = -(x^2 + 6x - 16)$

$= -((x+3)^2 - 9 - 16)$

$= 25 - (x+3)^2$

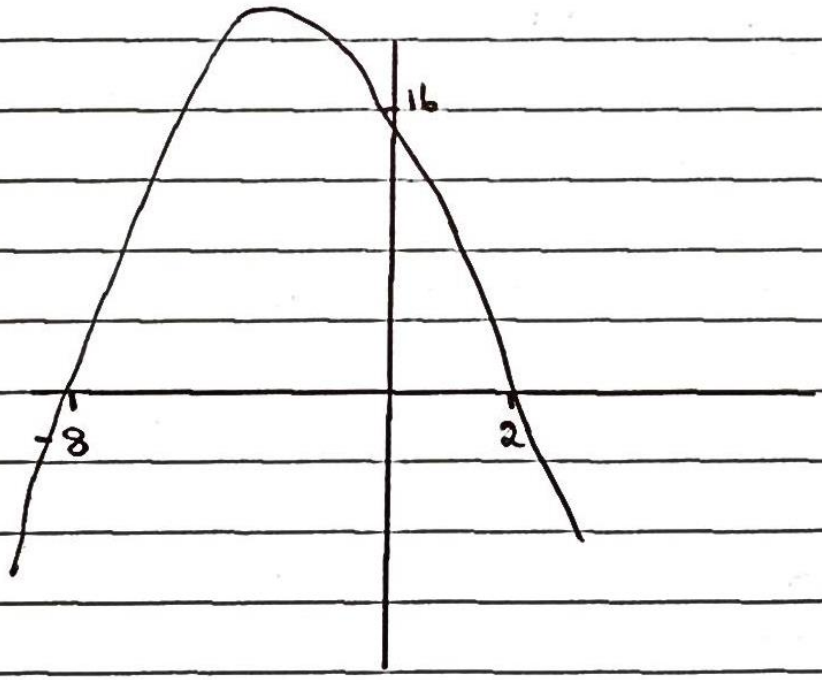
4aii. Max value is 25

4bi. $16 - 6x - x^2$

$(-x+2)(x+8)$

$= (2-x)(x+8)$

4bii. y intercept at $x = 0 \Rightarrow 16 - 6(0) - (0)^2 = 16$



5a. $p(x) = x^3 + cx^2 + dx + 3$

if $x+3$ a factor $p(-3) = 0$

$$p(-3) = (-3)^3 + c(-3)^2 + d(-3) + 3$$

$$0 = -27 + 9c - 3d + 3$$

$$24 = 9c - 3d$$

$$8 = 3c - d$$

5b. $p(2) = 65$

$$\Rightarrow \quad 65 = (2)^3 + c(2)^2 + d(2) + 3$$

$$65 = 8 + 4c + 2d + 3$$

$$54 = 4c + 2d$$

$$27 = 2c + d$$

5c. $8 = 3c - d$ ①

$27 = 2c + d$ ②

① + ② $\quad 35 = 5c \Rightarrow c = 7$

sub in ① $\quad 8 = 21 - d \Rightarrow d = 13$

6ai. $y = x^3 - x^2 - 5x + 7$ $\quad y = x + 7$ $\quad \ldots 1$

$x^3 - x^2 - 5x + 7 = x + 7$

$x^3 - x^2 - 6x = 0$

$x(x^2 - x - 6) = 0$

$x = 0$ at B so A & C satisfy $(x^2 - x - 6) = 0$

6aii. $x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0 \quad \Rightarrow \quad x = 3$ or -2

so at A $x = -2 \Rightarrow y = -2 + 7 = 5 \quad (-2, 5)$

at B $x = 3 \Rightarrow y = 3 + 7 = 10 \quad (3, 10)$

6b $\int x^3 - x^2 - 5x + 7 \, dx$

$= \frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{5}{2}x^2 + 7x + c$

6c. Area under curve $= \left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{5}{2}x^2 + 7x \right]_{-2}^{0}$

$= 0 - \left(4 + \frac{8}{3} - 10 + 14 \right) = 0 - \frac{52}{3} = \frac{52}{3}$

$\int_{-2}^{0} x + 7 \, dx = \left[\frac{1}{2}x^2 + 7x \right]_{-2}^{0}$

$= 0 - (2 - 14) = 12$

So $R = \frac{52}{3} - 12 = \frac{52}{3} - \frac{36}{3} = \frac{16}{3}$

7a. $x^2 + y^2 - 10x + 12y + 41 = 0$ $\quad A = (3, -2)$

$(x-5)^2 - 25 + (y+6)^2 - 36 + 41 = 0$

$(x-5)^2 + (y+6)^2 = 20$

7b. $C = (5, -6)$

7b. $R = \sqrt{20} = \sqrt{5 \times 4} = 2\sqrt{5}$

7c. m of $AC = \dfrac{-2 - -6}{3-5} = -2$

$\Rightarrow$ m of tangent $= +1/2$

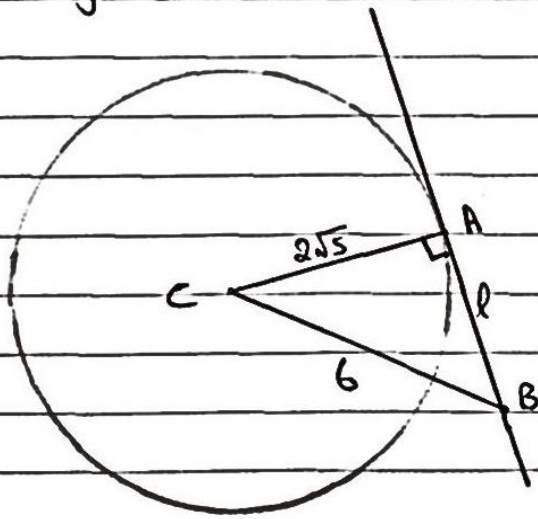
$y = \frac{1}{2}x + c$

$-2 = \frac{3}{2}x + c \quad \Rightarrow \quad c = -7/2$

$y = \frac{1}{2}x - \frac{7}{2}$

$2y = x - 7 \quad \Rightarrow \quad x - 2y = 7$

7d.



$6^2 = l^2 + (2\sqrt{5})^2$

$36 = l^2 + 20$

$\Rightarrow l^2 = 16$

$\Rightarrow AB = 4$

8a. $3(1-2x) - 5(3x+2) > 0$

$3 - 6x - 15x - 10 > 0$

$-21x > 7 \qquad \div -21$

$x < -1/3$

8b. $6x^2 \leq x + 12$

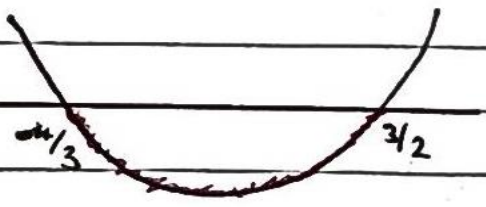
$6x^2 - x - 12 \leq 0$

$$x = \frac{1 \pm \sqrt{1^2 - 4(6)(-12)}}{2(6)}$$

$$= \frac{1 \pm 17}{12}$$

P.V. $= \frac{18}{12}, \frac{3}{2}$

or $\frac{-16}{12}, -4/3$



$$\Rightarrow -\frac{4}{3} \leq x \leq \frac{3}{2}$$