AQA

A Level

A Level Maths

AQA Core Maths C4 June 2014 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

AQA June 14 C4

1a.

$$x = \frac{t^2}{2} + 1 \qquad y = \frac{4}{t} - 1$$

$$\frac{dx}{dt} = t \qquad \frac{dy}{dt} = -\frac{4}{t^2}$$

$$\frac{dy}{dx} = \frac{-4/t^2}{t} = -\frac{4}{t^3}$$

when $t = 2$, $\dfrac{dy}{dx} = -\dfrac{4}{2^3} = -\dfrac{1}{2}$

1b.

$$y = \frac{4}{t} - 1$$

$$t = \frac{4}{y+1} \Rightarrow x = \frac{\left(\frac{4}{y+1}\right)^2}{2} + 1$$

$$x = \frac{1}{2}\left(\frac{4}{y+1}\right)^2 + 1$$

2a.

$$\frac{4x^3 - 2x^2 + 16x - 3}{2x^2 - x + 2} = Ax + \frac{B(4x-1)}{2x^2-x+2}$$

$$4x^3 - 2x^2 + 16x - 3 = Ax(2x^2 - x + 2) + B(4x-1)$$

$x = 0$; $\quad -3 = -B \Rightarrow B = 3$

$x = 1/4$; $\quad \dfrac{15}{16} = \dfrac{15}{32}A \Rightarrow A = 2$

$$\frac{4x^3 - 2x^2 + 16x - 3}{2x^2 - x + 2} = 2x + \frac{3(4x-1)}{2x^2-x+2}$$

2b. $\displaystyle\int 2x + \frac{3(4x-1)}{2x^2-x+2}\,dx$ $\qquad (-1, 2)$

$y = x^2 + 3\ln|2x^2-x+2| + c$

when $x = -1$, $y = 2$

$2 = 1 + 3\ln 5 + c \quad \Rightarrow \quad c = 1 - 3\ln 5$

$y = x^2 + 3\ln|2x^2-x+2| + 1 - 3\ln 5$

3a. $(1-4x)^{1/4} \approx 1 + \left(\frac{1}{4}\right)(-4x) + \frac{\left(\frac{1}{4}\right)\left(-\frac{3}{4}\right)}{2}(-4x)^2 + \ldots$

$= 1 - x - \frac{3}{2}x^2 + \ldots$

3b. $(2+3x)^{-3} = \left[2\left(1+\frac{3}{2}x\right)\right]^{-3}$

$= 2^{-3}\left(1+\frac{3}{2}x\right)^{-3}$

$= \frac{1}{8}\left(1+\frac{3}{2}x\right)^{-3}$

$= \frac{1}{8}\left(1 + (-3)\left(\frac{3}{2}x\right) + \frac{(-3)(-4)}{2}\left(\frac{3}{2}x\right)^2 + \ldots\right)$

$= \frac{1}{8}\left(1 - \frac{9}{2}x + \frac{27}{2}x^2 + \ldots\right)$

$= \frac{1}{8} - \frac{9}{16}x + \frac{27}{16}x^2 + \ldots$

3c. $\dfrac{(1-4x)^{1/4}}{(2+3x)^3} = \left(1 - x - \frac{3}{2}x^2\right)\left(\frac{1}{8} - \frac{9}{16}x + \frac{27}{16}x^2\right)$

$= \frac{1}{8} - \frac{9}{16}x + \frac{27}{16}x^2 - \frac{1}{8}x - \frac{9}{16}x^2 - \frac{3}{16}x^2 + \ldots$

$= \frac{1}{8} - \frac{11}{16}x + \frac{33}{16}x^2$

4a. when $t = 0$, $V = 5{,}000$

$V = Ap^t$

$5000 = Ap^0 \Rightarrow A = 5000$

4bi. in 2011, $t = 10$ $V = 25{,}000$

$25\,000 = 5000p^{10}$

$p^{10} = 5$

4bii. $V = 75{,}000$

$75{,}000 = 5000p^t$

$p^t = 15$

$t \ln p = \ln 15$

$t = \dfrac{\ln 15}{\ln p}$

from 4bi. $p^{10} = 5$

$10 \ln p = \ln 5$

$\ln p = \frac{1}{10} \ln 5$

$t = \dfrac{\ln 15}{\frac{1}{10} \ln 5}$

$= 16.83$ years

$=$ 16 years 10 months

so February 2018

4ci. $W = 2500q^t$ in April 1991 so $V = 5000p^{t-10}$

$5000p^{t-10} = 2500q^t$ (in 1991)

$2p^{t-10} = q^t$

$\ln(2p^{t-10}) = t \ln q$

$\ln 2 + (t-10) \ln p = t \ln q$

$$\ln 2 + t\ln p - 10\ln p = t\ln q$$

from 4bii. $10\ln p = \ln 5$

$$\ln 2 + t\ln p - \ln 5 = t\ln q$$

$$t\ln p - t\ln q = \ln 5 - \ln 2$$

$$t\left(\ln\left(\frac{p}{q}\right)\right) = \ln(5/2)$$

$$t = \frac{\ln(5/2)}{\ln(p/q)}$$

4cii. $p = 1.029q$

$$t = \frac{\ln\left(\frac{5}{2}\right)}{\ln\left(\frac{1.029q}{q}\right)} = \frac{\ln(5/2)}{\ln(1.029)}$$

$$= 32.05\ldots \text{ years}$$

April 1991 + 32.05 years

= April 2023

5ai.

$$3\sin x + 4\cos x = R\sin(x+\alpha)$$

$$3\sin x + 4\cos x = R(\sin x\cos\alpha + \cos x\sin\alpha)$$

$$R = \sqrt{3^2+4^2} = 5$$

$\sin x$: $3 = 5\cos\alpha$

$$\alpha = \cos^{-1}(3/5)$$

$$= 53.1^\circ \text{ (to 1dp)}$$

$$3\sin x + 4\cos x = 5\sin(x+53.1)$$

5aii.

$3\sin 2\theta + 4\cos 2\theta = 5$ $\qquad 0 < \theta < 360$

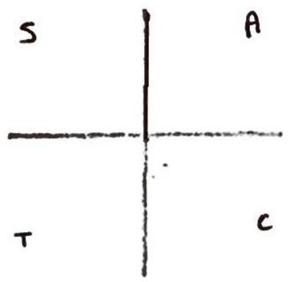
$5\sin(2\theta + 53.1) = 5$

$\sin(2\theta + 53.1) = 1$

Let $\phi = 2\theta + 53.1$

$\sin\phi = 1$ $\qquad 53.1 < \phi < 773.1$

P.V. $\phi = 90^\circ$



$\phi = 90^\circ, 450^\circ$

$2\theta + 53.1^\circ = 90^\circ \Rightarrow \theta = 18.4^\circ$

$2\theta + 53.1^\circ = 450^\circ \Rightarrow \theta = 198.4^\circ$

5bi.

$\tan 2\theta \tan\theta = 2$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\dfrac{2\tan^2\theta}{1-\tan^2\theta} = 2$

$\tan^2\theta = 1 - \tan^2\theta$

$2\tan^2\theta = 1$

5bii.

$\tan^2\theta = \dfrac{1}{2}$ $\qquad 0 \leq \theta \leq 180$

$\tan\theta = \pm\dfrac{\sqrt{2}}{2}$

$\tan\theta = \dfrac{\sqrt{2}}{2}$ or $\tan\theta = -\dfrac{\sqrt{2}}{2}$

$\theta = 35.3^\circ$

P.V. $\theta = -35.3 + 180^\circ$

$\theta = 144.7^\circ$

5ci. let $f(x) = 8x^3 - 4x + 1$

$f(1/2) = 8(1/2)^3 - 4(1/2) + 1$

$= 0$ $\therefore$ $2x-1$ is a factor

5cii. $4\cos 2\theta \cos\theta + 1$ $\qquad x = \cos\theta$

$4(2\cos^2\theta - 1)\cos\theta + 1$

$8\cos^3\theta - 4\cos\theta + 1$

$8x^3 - 4x + 1$

5ciii. $8x^3 - 4x + 1 = (2x-1)(4x^2 + 2x - 1) = 0$

if $2x - 1 = 0$

$x = 1/2$

$\cos\theta = 1/2$

$\theta = 60°$

so $4x^2 + 2x - 1 = 0$ $\qquad x = \cos 72$ must be a solution

$$x = \frac{-2 \pm \sqrt{2^2 - 4(4)(-1)}}{8}$$

$$x = \frac{-1 \pm \sqrt{5}}{4}$$

$$\cos x = \frac{-1 \pm \sqrt{5}}{4}$$

$\cos 72 > 0$ so want positive solution

$$\therefore \cos 72° = \frac{-1 + \sqrt{5}}{4}$$

6a. $l_1 \quad \underline{r} = \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$

$l_2 \quad \underline{r} = \begin{pmatrix} 7 \\ -8 \\ 6 \end{pmatrix} + m \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$

$P: \begin{pmatrix} 4 + (-1)(-1) \\ -5 + (-1)(3) \\ 3 + (-1)(1) \end{pmatrix} = \begin{pmatrix} 5 \\ -8 \\ 2 \end{pmatrix}$

$Q: \begin{pmatrix} 7 + 2(2) \\ -8 + 2(-3) \\ 6 + 2(1) \end{pmatrix} = \begin{pmatrix} 11 \\ -14 \\ 8 \end{pmatrix}$

$\overrightarrow{PQ} = \begin{pmatrix} 6 \\ -6 \\ 6 \end{pmatrix}$

$= 6 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad \therefore \ \overrightarrow{PQ} \ // \text{ to } \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

6bi. $\begin{pmatrix} 7+2m \\ -8-3m \\ 6+m \end{pmatrix} = \begin{pmatrix} 4-\lambda \\ -5+3\lambda \\ 3+\lambda \end{pmatrix} = \begin{pmatrix} 3 \\ b \\ c \end{pmatrix}$

$7 + 2m = 3 \Rightarrow m = -2$

$4 - \lambda = 3 \Rightarrow \lambda = 1$

check in $\underline{j}$ $\quad -8-3(-2) = -5+3(1) = b$

$-2 = -2 = b$ ✓

check in $\underline{k}$ $\quad 6+(-2) = \cancel{4\,5\,4}\ 3+1 = c$

$4 = 4 = c$ ✓

$R \ (3, -2, 4)$

6bii. Let l_3 be the line through P parallel to l_2

l_3 $\underline{r} = \begin{pmatrix} 5 \\ -8 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$

S lies on l_3 so $\overrightarrow{OS} = \begin{pmatrix} 5 + 2\mu \\ -8 - 3\mu \\ 2 + \mu \end{pmatrix}$ for some μ

$\overrightarrow{RS}$ $\perp$ to $\overrightarrow{PQ}$ $\therefore$ $\overrightarrow{RS} \cdot \overrightarrow{PQ} = 0$

$$\overrightarrow{RS} = \begin{pmatrix} 5 + 2\mu \\ -8 - 3\mu \\ 2 + \mu \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 + 2\mu \\ -6 - 3\mu \\ -2 + \mu \end{pmatrix}$$

so $\begin{pmatrix} 2 + 2\mu \\ -6 - 3\mu \\ -2 + \mu \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = 0$

$$2 + 2\mu - (-6 - 3\mu) + (-2 + \mu) = 0$$

$$2 + 2\mu + 6 + 3\mu - 2 + \mu = 0$$

$$6\mu = -6$$

$$\mu = -1$$

$$\overrightarrow{OS} = \begin{pmatrix} 5 + 2(-1) \\ -8 - 3(-1) \\ 2 + (-1) \end{pmatrix}$$

$$S = (3, -5, 1)$$

7ai.

$$\cos 2y + ye^{3x} = 2\pi$$

$$-2\sin 2y \frac{dy}{dx} + 3ye^{3x} + e^{3x}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(e^{3x} - 2\sin 2y\right) = -3ye^{3x}$$

$$\frac{dy}{dx} = \frac{-3ye^{3x}}{e^{3x} - 2\sin 2y}$$

7aii

at A, $x = \ln 2$, $y = \pi/4$

$$\frac{dy}{dx} = \frac{-3\left(\frac{\pi}{4}\right)e^{3\ln 2}}{e^{3\ln 2} - 2\sin(\pi/2)}$$

$$= -\pi$$

7b.

grad of normal $= \frac{1}{\pi}$ (since $\perp$)

$$y - \frac{\pi}{4} = \frac{1}{\pi}(x - \ln 2)$$

crosses y axis when $x = 0$

$$y - \frac{\pi}{4} = \frac{1}{\pi}(0 - \ln 2)$$

$$y = \frac{\pi}{4} - \frac{\ln 2}{\pi}$$

8a.

$$\frac{16x}{(1-3x)(1+x)^2} = \frac{A}{1-3x} + \frac{B}{1+x} + \frac{C}{(1+x)^2}$$

$$16x = A(1+x)^2 + B(1-3x)(1+x) + C(1-3x)$$

$x = -1$; $\quad -16 = 4C \quad \Rightarrow \quad C = -4$

$x = 1/3$; $\quad \frac{16}{3} = \frac{16}{9}A \quad \Rightarrow \quad A = 3$

$x = 0$;

$$0 = A + B + C$$

$$0 = 3 + B - 4 \quad \Rightarrow \quad B = 1$$

$$\frac{16x}{(1-3x)(1+x)^2} = \frac{3}{1-3x} + \frac{1}{1+x} - \frac{4}{(1+x)^2}$$

8b.

$$\frac{dy}{dx} = \frac{16x\,e^{2y}}{(1-3x)(1+x)^2}$$

$$\int e^{-2y}\,dy = \int \frac{16x}{(1-3x)(1+x)^2}\,dx$$

$$\int e^{-2y}\,dy = \int \frac{3}{1-3x} + \frac{1}{1+x} - \underline{\frac{4}{(1+x)^2}}\,dx$$

$$\int \frac{4}{(1+x)^2}\,dx : \quad \frac{d}{dx}\,k(1+x)^{-1} = 4(1+x)^{-2}$$

$$-k(1+x)^{-2} = 4(1+x)^{-2}$$

$$\therefore \quad k = -4$$

so $\int \frac{4}{(1+x)^2}\,dx = -4(1+x)^{-1}$

$$\int e^{-2y} = \int \frac{3}{1-3x} + \frac{1}{1+x} \, dx \quad - \left(-4(1+x)^{-1}\right)$$

$$-\frac{1}{2}e^{-2y} = -\ln|1-3x| + \ln|1+x| + 4(1+x)^{-1} + c$$

when $x=0$, $y=0$

$$-\frac{1}{2}e^{0} = -\ln 1 + \ln 1 + \frac{4}{1} + c$$

$$-\frac{1}{2} = 4 + c \quad \Rightarrow \quad c = -\frac{9}{2}$$

$$-\frac{1}{2}e^{-2y} = -\ln|1-3x| + \ln|1+x| + \frac{4}{1+c} - \frac{9}{2}$$