Visit http://www.mathsmadeeasy.co.uk/ for more fantastic resources.

AQA

A Level

A Level Maths

AQA Core Maths C3 June 2014 Model Solutions

Name:

MME
Mathsmadeeasy.co.uk

Total Marks:

Jun 14 C3 - AQA

1. $\int_0^{\pi} x^{1/2} \sin x \, dx$

Simpson's 4 strips let $y(x) = x^{1/2} \sin x$

$y(0) = 0$

$y(\pi/4) = 0.6266570687$

$y(\pi/2) = 1.253314137$

$y(3\pi/4) = 1.085401882$

$y(\pi) = 0$ $h = \frac{\pi - 0}{4} = \frac{\pi}{4}$

$$\int_0^{\pi} x^{1/2} \sin x \, dx \approx \frac{1}{3} \times \frac{\pi}{4} \left\{ (0) + 4(0.6266... + 1.0854...) + 2(1.253...) \right\}$$

$= 2.449097623$

$= 2.449$ to 4 s.f.

2a. $y = 2\ln(2e - x) = \ln(2e-x)^2$

$\frac{dy}{dx} = \frac{-2(2e-x)}{(2e-x)^2}$

2b. at $x = e$, $y = 2$

m at $(e, 2)$ $\frac{dy}{dx} = \frac{-2e}{e^2} = \frac{-2}{e}$

$\Rightarrow$ m of normal $= \frac{e}{2}$

$y - 2 = \frac{e}{2}(x - e)$

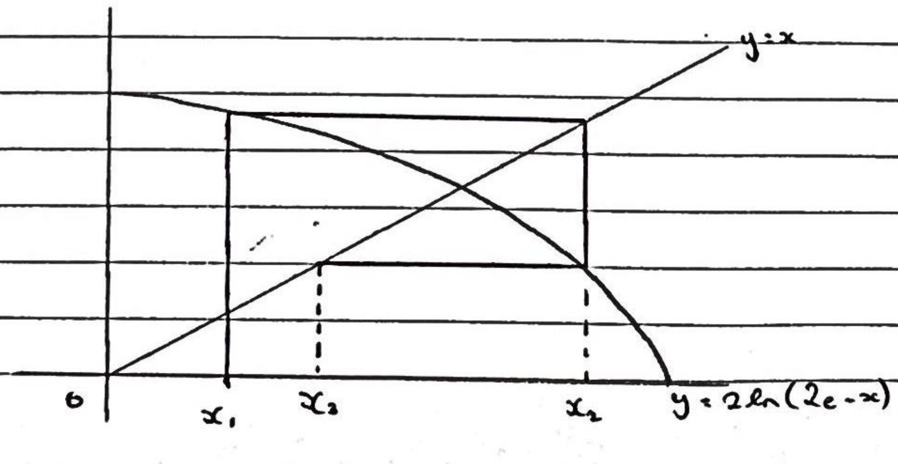
2ci. let $f(x) = 2\ln(2e - x) - x$

$f(1) = 1.97976...$
$f(3) = -1.2188...$

change of sign $\Rightarrow$ $\alpha \in [1, 3]$

2cii. $x_{n+1} = 2\ln(2e - x_n)$

$x_1 = 1$
$x_2 = 2.97976... = 2.980$ to 3dp
$x_3 = 1.79772... = 1.798$ to 3dp.



3ai. $(x^2+1)^{5/2}$

$$\frac{d}{dx}(x^2+1)^{5/2} = \frac{5}{2} \cdot 2x\,(x^2+1)^{3/2}$$

$$= 5x\,(x^2+1)^{3/2}$$

3aii. $y = e^{2x}(x^2+1)^{5/2}$

$$\frac{dy}{dx} = 2e^{2x}(x^2+1)^{5/2} + 5xe^{2x}(x^2+1)^{3/2}$$

when $x = 0$, $\frac{dy}{dx} = 2$

3b. $y = \frac{4x-3}{x^2+1}$ $\quad f = 4x-3 \quad g = x^2+1$

$f' = 4 \quad g' = 2x$

$$\frac{dy}{dx} = \frac{4(x^2+1) - 2x(4x-3)}{(x^2+1)^2}$$

$$4x^2 + 4 - 8x^2 + 6x = 0$$

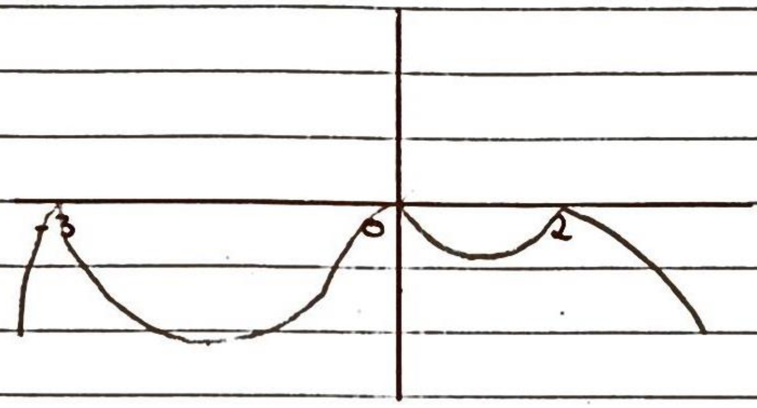
$$4x^2 - 6x - 2 = 0$$

$$2x^2 - 3x - 1 = 0$$

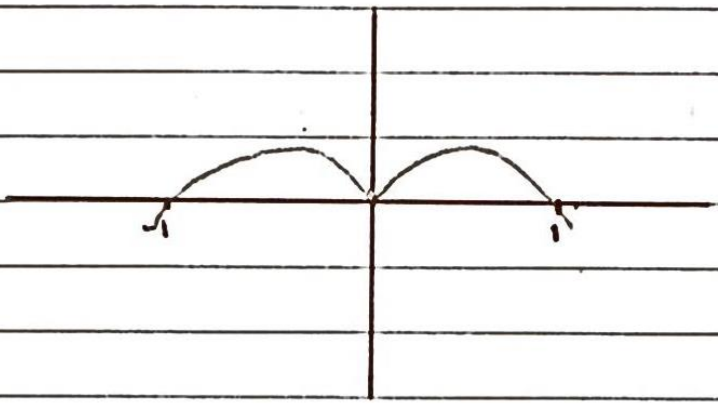
$$(2x+1)(x-1) = 0$$

$\Rightarrow x = 1$ or $-1/2$

4a.



4b.



4ci. $f(x) \rightarrow f(2x)$ stretch x axis s.f. $1/2$

$f(2x) \rightarrow f(2(x+1))$ translation -1 in positive x direction

4cii. $(4 \times 1/2) - 1 = 1$

so $(1, -3)$

5a. $f(x) = x^2 - 6x + 5$ for $x \geqslant 3$

$g(x) = |x - 6|$ $\forall x \in \mathbb{R}$

$f(x) = (x-3)^2 - 9 + 5$

$= (x-3)^2 - 4$

$\Rightarrow f(x) \geqslant -4$

5b $y = (x-3)^2 - 4$

$\pm\sqrt{y+4} = x - 3$

$x = 3 \pm \sqrt{y+4}$

so $f^{-1}(x) = 3 + \sqrt{x+4}$

5ci. $gf(x) = |x^2 - 6x + 5 - 6| = |x^2 - 6x - 1|$

5cii. $|x^2 - 6x - 1| = 6$

$x^2 - 6x - 1 = 6$ or $x^2 - 6x - 1 = -6$

$\Rightarrow x^2 - 6x - 7 = 0$ $\Rightarrow x^2 - 6x + 5 = 0$

$(x - 7)(x+1) = 0$ $(x \cancel{+} -1)(x-5) = 0$

$\Rightarrow x = 7, -1, 1,$ or 5

but $f(x)$ only true for $x \geqslant 3$

$\Rightarrow x = 5$ or 7

S C
-C -S

6a. $\int x^2 \sin 2x \, dx$

$u = x^2 \quad v' = \sin 2x$

$u' = 2x \quad v = -\frac{1}{2}\cos 2x$

$= -\frac{1}{2}x^2\cos 2x + \underbrace{\int x\cos 2x \, dx}$

$u = x \quad v' = \cos 2x$

$u' = 1 \quad v = \frac{1}{2}\sin 2x$

$= \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x \, dx$

$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$

$\Rightarrow \int x^2\sin 2x \, dx = -\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$

6b. $V = \pi\int_0^{\pi/2} y^2 \, dx = \pi\int_0^{\pi/2} x^2\sin 2x \, dx$

$= \pi\left[-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right]_0^{\pi/2}$

$= \pi\left[\left(\frac{\pi^2}{8} + 0 - \frac{1}{4}\right) - \left(\frac{1}{4}\right)\right]$

$= \frac{\pi^3}{8} - \frac{\pi}{2}$

7. $\displaystyle\int_0^1 \frac{x^5}{3-x^3}\,dx$

$u = 3 - x^3 \qquad du = -3x^2\,dx$

$x^3 = 3 - u$

$x = (3-u)^{1/3}$

$dx = -\frac{1}{3}(3-u)^{-2/3}\,du$

$$-\frac{1}{3}\int_3^2 \frac{(3-u)^{5/3}}{u}\cdot(3-u)^{-2/3}\,du$$

x	1	0
u	2	3

$$= -\frac{1}{3}\int_3^2 3u^{-1} - 1\,du$$

$$= -\frac{1}{3}\Big[3\ln u - u\Big]_3^2$$

$$= -\frac{1}{3}\Big((3\ln 2 - 2) - (3\ln 3 - 3)\Big)$$

$$= -\frac{1}{3}\Big[\ln(8/27) + 1\Big]$$

$$= -\frac{1}{3}\ln(8/27) - \frac{1}{3}$$

$$= \ln(3/2) - \frac{1}{3}$$

8a. $\dfrac{1-\sin x}{\cos x} + \dfrac{\cos x}{1-\sin x} \equiv 2\sec x$

LHS $\dfrac{(1-\sin x)^2 + \cos^2 x}{\cos x\,(1-\sin x)}$

$\dfrac{1 - 2\sin x + \sin^2 x + \cos^2 x}{\cos x\,(1-\sin x)}$ use $\cos^2 x + \sin^2 x \equiv 1$

$\dfrac{2 - 2\sin x}{\cos x\,(1-\sin x)} = \dfrac{2(1-\sin x)}{\cos x\,(1-\sin x)}$

$= \dfrac{2}{\cos x} = \text{RHS}$

8b. $2\sec x = \tan^2 x - 2$ use $\tan^2 x \equiv \sec^2 x - 1$

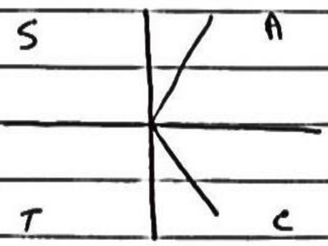
$2\sec x = \sec^2 x - 3$

$\sec^2 x - 2\sec x - 3 = 0$

$(\sec x + 1)(\sec x - 3) = 0$

$\dfrac{1}{\cos x} = -1$ or $\dfrac{1}{\cos x} = +3$

$\Rightarrow \cos x = 180$ $\Rightarrow \cos x = \cancel{109.471}$ 71°



$x = 71^\circ, 289^\circ$ or 180°

8c. $2\theta - 30^\circ = 71^\circ \Rightarrow \theta = 50.5^\circ$

$2\theta - 30^\circ = 180^\circ \Rightarrow \theta = 105^\circ$

$2\theta - 30^\circ = 289^\circ \Rightarrow \theta = 160^\circ$