AQA

A Level

A Level Maths

AQA Core Maths C2 June 2014 Model Solutions

Name:

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Total Marks:

Jun 2014 AQA C2

1a. $A = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(5)(12)\sin 47^\circ$

$= 21.94\ldots$

$= 22\ \text{cm}^2$

1b. $a^2 = b^2 + c^2 - 2bc\cos A$

$a = 5^2 + 12^2 - 2(5)(12)\cos 47^\circ$

$a = 9.33596\ldots$

$= 9.3\ \text{cm}$

2a $\int 1 + 3x^{1/2} + x^{3/2}\,dx = x + 2x^{3/2} + \frac{2}{5}x^{5/2} + c$

2bi. $(1+y)^3 = (1+y)(1+2y+y^2) = 1 + 3y + 3y^2 + y^3$

$\Rightarrow n = 3$

2bii. $(1+\sqrt{x})^3 = 1 + 3x^{1/2} + 3x + x^{3/2}$

2c. $\int_0^1 1 + 3x^{1/2} + 3x + x^{3/2}\,dx$

$= \left[x + 2x^{3/2} + \frac{3}{2}x^2 + \frac{2}{5}x^{5/2}\right]$

$\frac{49}{10} - 0$

$= \frac{49}{10}$

3a. $a = 54$, $r = 8/9$

$$S_\infty = \frac{54}{1 - 8/9} = 486$$

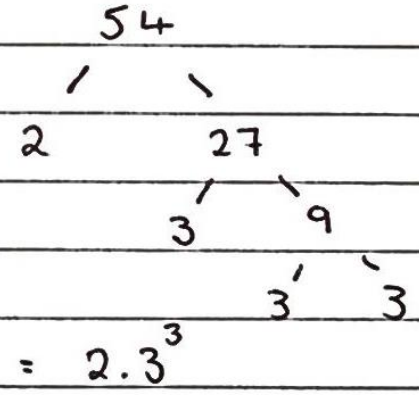
3b. $u_2 = 54\left(\frac{8}{9}\right)^1 = 48$

3c. $u_{12} = 54 \cdot \left(\frac{8}{9}\right)^{11}$

$$= \frac{54 \cdot 8^{11}}{9^{11}}$$

$8^{11} = (2^3)^{11} = 2^{33}$

$9^{11} = (3^2)^{11} = 3^{22}$



$54 = 2.3^3$

$$u_{12} = \frac{2.3^3.2^{33}}{3^{22}}$$

$$= \frac{2^{34}}{3^{19}}$$

4a. $y = x^{-2} + 4x$

$$\frac{dy}{dx} = -2x^{-3} + 4$$

4b. at P, $x = -1 \Rightarrow \frac{dy}{dx} = -2(-1)^{-3} + 4$

$$= 6$$

$\Rightarrow$ m of normal $= -1/6$

$$(y+3) = -\frac{1}{6}(x+1)$$

4c. $y = -12x \Rightarrow \frac{dy}{dx} = -12$

$$-12 = -2x^{-3} + 4$$

$$2x^{-3} = 16$$

$$x^{-3} = 8$$

$$x = 1/2$$

when $x = 1/2$ $y = 6$

$$y - 6 = -12(x - 1/2)$$

5. $A = \frac{1}{2}r^2\theta$

length $= r\theta$

$4r\theta = 2r + r\theta$

$3r\theta = 2r$

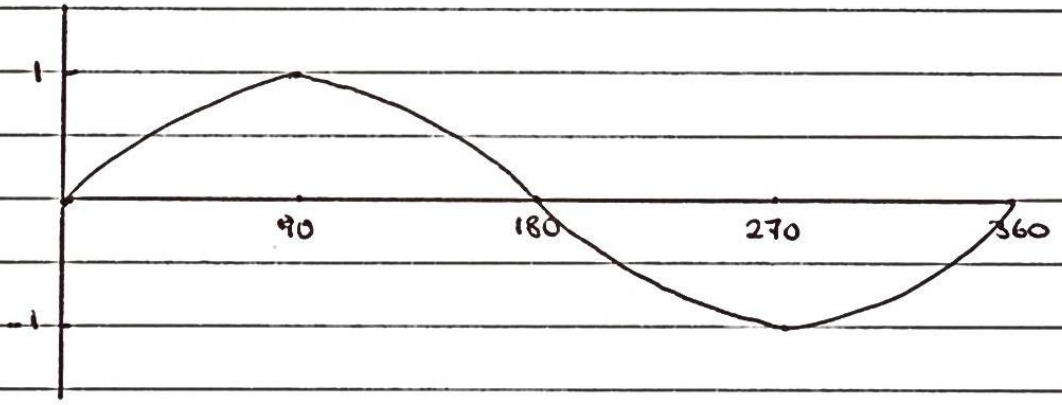
$3\theta = 2$

$\theta = 2/3$

$12 = \frac{1}{2}r^2 \cdot (2/3)$

$16 = r^2 \quad \Rightarrow \quad r = 4$

6a.



6b. $y = \sin x \rightarrow y = \sin 5x$ stretch s.f. 1/5 in x direction

6c. $y = \sin x \rightarrow y = \sin(5x + 10)$

$y = \sin(5(x+2))$

translation 2° in negative x direction

7a. $\dfrac{\cos^2 x + 4\sin^2 x}{1 - \sin^2 x} = 7$

use $\sin^2 x \equiv 1 - \cos^2 x$

$$\frac{\cos^2 x + 4\sin^2 x}{\cos^2 x} = 7$$

$$\frac{4\sin^2 x}{\cos^2 x} = 6$$

$$4\tan^2 x = 6$$

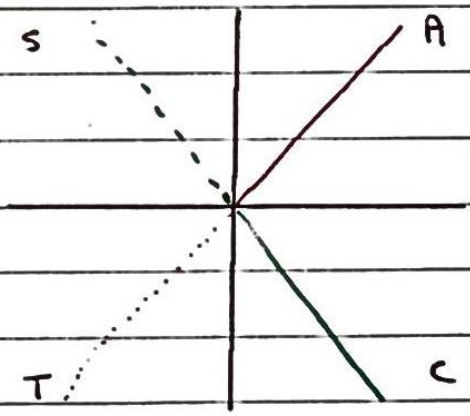
$$\tan^2 x = 3/2$$

7b. $\tan^2 2\theta = \dfrac{3}{2}$ $\quad 0 < \theta < 180$

Let $2\theta = \alpha$ $\quad 0 < \alpha < 360$

$$\alpha = \pm\sqrt{\frac{3}{2}}$$

P.V. ± 50.768



$\alpha = 50.77^\circ$

$= 230.77^\circ$

$= 309.23^\circ$

$= 129.23^\circ$

$\theta = 25^\circ, 115^\circ, 155^\circ, 65^\circ$

8a. $S_5 = \frac{1}{2}(5)[2a + 4d] = 575$

$2a + 4d = 230$

$a + 2d = 115$

8b. $u_{10} = a + 9d = 87$

$7d = 87 - 115$

$7d = -28$

$d = -4$

8c. $d = -4 \quad a = 123$

$u_k = 123 - 4(k-1) > 0$

$123 - 4k + 4 > 0$

$4k < 127$

$k < 31.75$

$u_{k+1} = 123 - 4k < 0$

$4k < 123$

$k < 30.75$

$\Rightarrow \quad k = 31$

$S_{31} = \frac{31}{2}[2(123) + 30(-4)]$

$= 1953$

9a. $y = 3 \times 12^x$

$6 = 3 \times 12^k$

$2 = 12^k$

$\ln 2 = k \ln 12$

$k = \dfrac{\ln 2}{\ln 12} = 0.27894\ldots$

$= 0.279$ to 3 s.f.

9b. $\displaystyle\int_0^{1.5} 3 \times 12^x \, dx$

3 strips $\Rightarrow$ $h = \dfrac{1.5 - 0}{3} = 0.5$

x_0	y_0
0	$3 \times 12^0 = 3$
0.5	$3 \times 12^{0.5}$
1	$3 \times 12 = 36$
1.5	$3 \times 12^{1.5}$

$\dfrac{1}{2} \cdot \dfrac{1}{2} \left\{ (3 + 3 \times 12^{1.5}) + 2(36 + 3 \times 12^{0.5}) \right\}$

$= 55.123\ldots$

$= 55$ to 2 s.f.

9c. $y = 3 \times 12^x \longrightarrow y = 3 \times 12^{(-1)}$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$y = 3 \times 12^x \longrightarrow y = 3 \times 12^x + p$ $\begin{bmatrix} 0 \\ p \end{bmatrix}$

$0 = 3 \times 12^{-1} + p$

$\Rightarrow p = -1/4$

9d. $y = 2^{(2-x)}$ $\quad y = 3 \times 12^{x}$

$$2^{(2-x)} = 3 \times 12^{x}$$

$$(2-x)\log_2 2 = \log_2 3 + x\log_2 12$$

$$(2-x)\log_2 2 = \log_2 3 + x(\log_2 4 + \log_2 3)$$

$\log_2 2 = 1$, $\log_2 4 = 2$

$$2 - x = \log_2 3 + 2x + x\log_2 3$$

$$2 - \log_2 3 = 3x + x\log_2 3$$

$$2 - \log_2 3 = x(3 + \log_2 3)$$

$$x = \frac{2 - \log_2 3}{3 + \log_2 3}$$