AQA

A Level

A Level Maths

AQA Core Maths C4 June 2013 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

AQA June 13 C4

1ai. $$\frac{5-8x}{(2+x)(1-3x)} = \frac{A}{2+x} + \frac{B}{1-3x}$$

$$5-8x = A(1-3x) + B(2+x)$$

$x = -2$; $21 = 7A \Rightarrow A = 3$

$x = 1/3$; $7/3 = 7/3 B \Rightarrow B = 1$

$$\frac{5-8x}{(2+x)(1-3x)} = \frac{3}{2+x} + \frac{1}{1-3x}$$

1aii. $$\int_{-1}^{0} \frac{3}{2+x} + \frac{1}{1-3x}\,dx$$

$$= \left[3\ln|2+x| - \frac{1}{3}\ln|1-3x|\right]_{-1}^{0}$$

$$= \left(3\ln 2 - \frac{1}{3}\ln 1\right) - \left(3\ln 1 - \frac{1}{3}\ln 4\right)$$

$$= 3\ln 2 + \frac{1}{3}\ln 4$$

$$= 3\ln 2 + \frac{2}{3}\ln 2$$

$$= \frac{11}{3}\ln 2$$

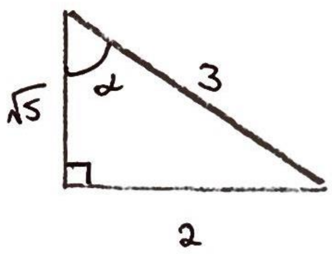
1bi. $(-6x^2) \div (-3x^2) = 2 \Rightarrow c = 2$

$$\int_{-1}^{0} \frac{9-18x-6x^2}{2-5x-3x^2}\,dx = \int_{-1}^{0} 2\,dx + \int_{-1}^{0} \frac{5-8x}{(2+x)(1-3x)}\,dx$$

$$= [2x]_{-1}^{0} + \frac{11}{3}\ln 2$$

$$= 2 + \frac{11}{3}\ln 2$$

2ai. $\tan\alpha = \frac{2}{\sqrt{5}}$ $\tan\beta = \frac{1}{2}$



$\text{hyp} = \sqrt{2^2 + (\sqrt{5})^2} = 3$

$\sin\alpha = \frac{\text{opp}}{\text{hyp}} = \frac{2}{3}$

$\cos\alpha = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{5}}{3}$

2aii.

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$= 2 \cdot \frac{2}{3} \cdot \frac{\sqrt{5}}{3}$$
$$= \frac{4}{9}\sqrt{5}$$

2b

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$\text{hyp} = \sqrt{2^2+1^2} = \sqrt{5}$

$\sin\beta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{5}}$

$\cos\beta = \frac{\text{adj}}{\text{hyp}} = \frac{2}{\sqrt{5}}$

$$\cos(\alpha-\beta) = \frac{\sqrt{5}}{3} \cdot \frac{2}{\sqrt{5}} + \frac{2}{3} \cdot \frac{1}{\sqrt{5}}$$
$$= \frac{2}{3} + \frac{2}{3\sqrt{5}}$$
$$= \frac{2}{3} + \frac{2\sqrt{5}}{15}$$
$$= \frac{2}{15}(5 + \sqrt{5})$$

3a.

$$(1+6x)^{-\frac{1}{3}} \approx 1 + (-\tfrac{1}{3})(6x) + \frac{(-\frac{1}{3})(-\frac{4}{3})}{2}(6x)^2 + \dots$$

$$= 1 - 2x + 8x^2 + \dots$$

3bi.

$$(27+6x)^{-1/3} = \left[27(1 + \tfrac{2}{9}x)\right]^{-1/3} = 27^{-1/3}(1+\tfrac{2}{9}x)^{-1/3}$$

$$= \frac{1}{3}\left(1 + (-\tfrac{1}{3})(\tfrac{2}{9}x) + \frac{(-\frac{1}{3})(-\frac{4}{3})}{2}(\tfrac{2}{9}x)^2 + \dots\right)$$

$$= \frac{1}{3}\left(1 - \frac{2}{27}x + \frac{8}{729}x^2 + \dots\right)$$

$$= \frac{1}{3} - \frac{2}{81}x + \frac{8}{2187}x^2 + \dots$$

3bi.

$$\sqrt[3]{\frac{2}{7}} = 2 \cdot 28^{-1/3} \quad : \quad 27 + 6x = 28$$

$$x = 1/6$$

$$= 2\left(\frac{1}{3} - \frac{2}{81}\left(\frac{1}{6}\right) + \frac{8}{2187}\left(\frac{1}{6}\right)^2\right)$$

$$= 0.658639 \quad (6dp)$$

4a.

$$x = 8e^{-2t} - 4 \qquad y = 2e^{2t} + 4$$

$$\frac{dx}{dt} = -16e^{-2t} \qquad \frac{dy}{dt} = 4e^{2t}$$

$$\frac{dy}{dt} = \frac{4e^{2t}}{-16e^{-2t}}$$

$$= -\frac{1}{4}e^{4t}$$

4bi.

when $t = \ln 2$, $\frac{dy}{dt} = -\frac{1}{4}e^{4\ln 2}$

$$= -4$$

4bii. when $t = \ln 2$ $\quad x = 8e^{-2\ln 2} - 4 \quad y = 2e^{2\ln 2} + 4$

$$= -2 \qquad\qquad = 12$$

$P(-2, 12)$

4biii. grad of normal $= \frac{1}{4}$ (since $\perp$)

$$y - 12 = \frac{1}{4}(x+2)$$

when $y = 0$,
$$-12 = \frac{1}{4}(x+2)$$
$$-48 = x + 2$$
$$x = -50 \qquad \therefore \quad (-50, 0)$$

4c.
$$xy + 4y - 4x = k$$

$$xy = (8e^{-2t} - 4)(2e^{2t} + 4)$$
$$= 16 + 32e^{-2t} - 8e^{2t} - 16$$
$$= 32e^{-2t} - 8e^{2t}$$

$$4y = 4(2e^{2t} + 4) = 8e^{2t} + 16$$

$$4x = 4(8e^{-2t} - 4) = 32e^{-2t} - 16$$

$$\cancel{32e^{-2t}} - \cancel{8e^{2t}} + \cancel{8e^{2t}} + 16 - (\cancel{32e^{-2t}} - 16)$$
$$16 - -16$$
$$= 32$$

$$\therefore \quad k = 32$$

5a.

$f(x) = 4x^3 - 11x - 3$

$f(-3/2) = 4(-3/2)^3 - 11(-3/2) - 3$

$= -\frac{27}{2} + \frac{33}{2} - 3$

$= 0$

$\therefore$ $(2x+3)$ is a factor

5b.

$$\begin{array}{r} 2x^2 - 3x - 1 \\ 2x+3 \overline{\smash{)}\, 4x^3 + 0x^2 - 11x - 3} \\ \underline{4x^3 + 6x^2} \qquad\qquad \\ -6x^2 - 11x \qquad \\ \underline{-6x^2 - 9x} \qquad \\ -2x - 3 \\ \underline{-2x - 3} \\ 0 \end{array}$$

$\therefore$ $f(x) = (2x+3)(2x^2 - 3x - 1)$

5ci.

$2\cos 2\theta \sin\theta + 9\sin\theta + 3 = 0$ $\qquad x = \sin\theta$

$2(1 - 2\sin^2\theta)\sin\theta + 9\sin\theta + 3 = 0$

$2\sin\theta - 4\sin^3\theta + 9\sin\theta + 3 = 0$

$4\sin^3\theta - 11\sin\theta - 3 = 0$

$4x^3 - 11x - 3 = 0$

5cii.

$(2x+3)(2x^2 - 3x - 1) = 0$

$2x + 3 = 0$ or $(2x^2 - 3x - 1) = 0$

$x = -3/2$

$\sin\theta \neq -3/2$

no solutions

$x = \frac{3 \pm \sqrt{3^2 - 4(2)(-1)}}{4}$

$= \frac{3 \pm \sqrt{17}}{4}$

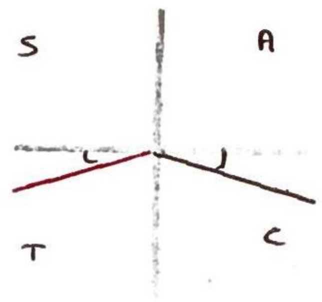
$$\sin\theta = \frac{3 \pm \sqrt{17}}{4}$$

$$\sin\theta \neq \frac{3+\sqrt{17}}{4}$$

No Solutions

$$\sin\theta = \frac{3-\sqrt{17}}{4}$$

P.V. $\theta = -16.31°$



$\theta = 344°, 196°$ (nearest degree)

6a. A $(3, -2, 4)$ B $(1, -5, 6)$ C $(-4, 5, -1)$

$$l: \underline{r} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ -7 \\ 5 \end{pmatrix}$$

$3 + 7\lambda = -4 \qquad \lambda = -1$

$-2 - 7\lambda = 5 \qquad \lambda = -1$

$4 + 5\lambda = -1 \qquad \lambda = -1$

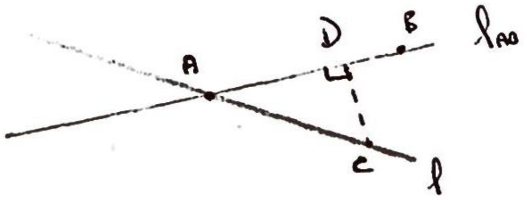
$\therefore$ C lies on l

6b. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$$= \begin{pmatrix} 1 \\ -5 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$$

$$l_{AB} \quad \underline{r} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} + m \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$$

6c.



D lies on l_{AB} so $\overrightarrow{OD} = \begin{pmatrix} 3-2m \\ -2-3m \\ 4+2m \end{pmatrix}$ for some m.

$\overrightarrow{CD} \perp$ to $\overrightarrow{AB} \iff \overrightarrow{CD} \cdot \overrightarrow{AB} = 0$

$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$

$= \begin{pmatrix} 3-2m \\ -2-3m \\ 4+2m \end{pmatrix} - \begin{pmatrix} -4 \\ 5 \\ -1 \end{pmatrix} = \begin{pmatrix} 7-2m \\ -7-3m \\ 5+2m \end{pmatrix}$

$\overrightarrow{CD} \cdot \overrightarrow{AB} = 0$

so $-2(7-2m) - 3(-7-3m) + 2(5+2m) = 0$

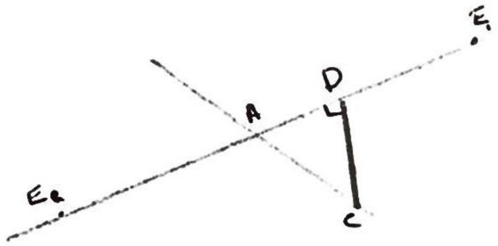
$-14 + 4m + 21 + 9m + 10 + 4m = 0$

$17m = -17$

$m = -1$

sub $m = -1$ into $\overrightarrow{OD} = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$ D at $(5,1,2)$

6d.



$\overrightarrow{AD} = \frac{1}{3}\overrightarrow{AE}$

$\overrightarrow{AD} = \overrightarrow{AA} - \overrightarrow{OD}$

$\begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$

$\overrightarrow{AD} = \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix}$

$\overrightarrow{DA} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$

$\overrightarrow{OE_2} = \overrightarrow{OA} + 3\cancel{\overrightarrow{AB}}\,DA$

$= \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} + 3\begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix} \Rightarrow$ E at $(-3,-11,10)$

$$\overrightarrow{OE_1} = \overrightarrow{OA} + 3\overrightarrow{AD}$$

$$= \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} + 3\begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} \Rightarrow E \text{ at } (9, 7, -2)$$

7. Max value of $\cos(kt) = 1$ so max $= 1.3 \Rightarrow a = 1.3$

repeats every $2\pi \Rightarrow k = \frac{2\pi}{12}$

so $\frac{dh}{dt} = 1.3\cos\left(\pi/6\right)$

8a. $\int t\cos\left(\frac{\pi}{4}t\right)\,dt$ Parts: $u = t$, $u' = 1$, $v' = \cos\left(\frac{\pi}{4}t\right)$, $v = \frac{4}{\pi}\sin\left(\frac{\pi}{4}t\right)$

$$= \frac{4t}{\pi}\sin\left(\frac{\pi}{4}t\right) - \int \frac{4}{\pi}\sin\left(\frac{\pi}{4}t\right)\,dt$$

$$= \frac{4t}{\pi}\sin\left(\frac{\pi}{4}t\right) + \frac{16}{\pi^2}\cos\left(\frac{\pi}{4}t\right) + c$$

8b. $$\frac{dx}{dt} = \frac{t\cos\left(\frac{\pi}{4}t\right)}{32x}$$

$$\int 32x\,dx = \int t\cos\left(\pi/4\,t\right)\,dt$$

$$16x^2 = \frac{4t}{\pi}\sin\left(\frac{\pi}{4}t\right) + \frac{16}{\pi^2}\cos\left(\frac{\pi}{4}t\right) + c$$

when $t = 0$, $x = 4 \Rightarrow 256 = \frac{16}{\pi^2} + c$ $\quad c = 256 - \frac{16}{\pi^2}$

$$16x^2 = \frac{4t}{\pi}\sin\left(\frac{\pi}{4}t\right) + \frac{16}{\pi^2}\cos\left(\frac{\pi}{4}t\right) + 256 - \frac{16}{\pi^2}$$

$$x = \sqrt{\frac{\frac{4}{\pi}t\sin\left(\frac{\pi}{4}t\right) + \frac{16}{\pi^2}\cos\left(\pi/4\,t\right) + 256 - \frac{16}{\pi^2}}{16}}$$

when $t = 45$,

$$x = \sqrt{\frac{\frac{4}{\pi}(45)\sin\left(\frac{45\pi}{4}\right) + \frac{16}{\pi^2}\cos\left(\frac{45\pi}{4}\right) + 256 - \frac{16}{\pi^2}}{16}}$$

$= 3.65\text{m}$ (nearest cm)