

AQA

A Level

A Level Maths

**AQA Core Maths C3 June 2013
Model Solutions**

Name:



Mathsmadeeasy.co.uk

Total Marks:

AQA

June 13

C3

1a. $|2x-3| = |x|$

$$2x-3 = x$$

$$x = 3$$

or $2x-3 = -x$

$$3x = 3$$

$$x = 1$$

1b. $x \leq 1, x \geq 3$

Since $y = |2x-3|$ is above $y = |x|$ when $x \leq 1$ and $x \geq 3$

2a. $y = x^4 \tan 2x$

$$x^4 \rightarrow 4x^3$$

$$\tan 2x \rightarrow 2 \sec^2 2x$$

$$\frac{dy}{dx} = 4x^3 \tan 2x + 2x^4 \sec^2 2x$$

2b. $y = \frac{x^2}{x-1}$

$$f = x^2$$

$$f' = 2x$$

$$g = x-1$$

$$g' = 1$$

$$\frac{dy}{dx} = \frac{2x(x-1) - x^2(1)}{(x-1)^2}$$

when $x = 3$, $\frac{dy}{dx} = \frac{2(3)(3-1) - (3)^2}{(3-1)^2}$

$$= \frac{3}{4}$$

3a. $e^{-x} - 2 + \sqrt{x} = 0$

$$f(3) = -0.218162 \dots$$

$$f(4) = 0.0183156 \dots$$

change of sign $\therefore 3 < \alpha < 4$

3b.

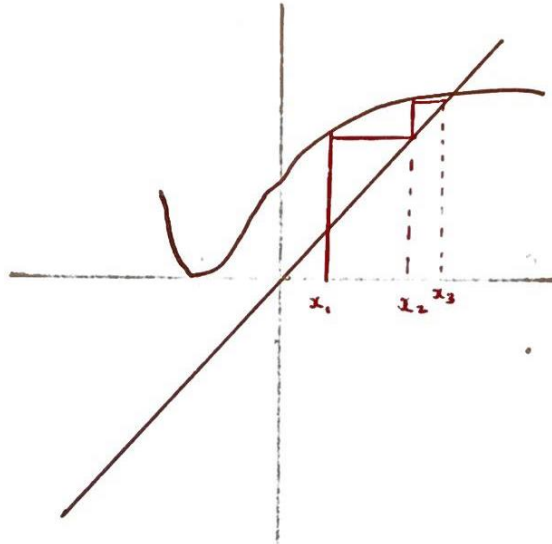
$$x_{n+1} = (2 - e^{-x_n})^2$$

$$x_1 = 3.5$$

$$x_2 = 3.880$$

$$x_3 = 3.918$$

3c.



4.

$$8 \sec x - 2 \sec^2 x = \tan^2 x - 2$$

$$0 < x < 2\pi$$

use $\tan^2 x \equiv \sec^2 x - 1$

$$\sec^2 x \equiv \tan^2 x + 1$$

$$8 \sec x - 2 \sec^2 x = (\sec^2 x - 1) - 2$$

$$3 \sec^2 x - 8 \sec x - 3 = 0$$

$$(3 \sec x + 1)(\sec x - 3) = 0$$

$$\therefore \sec x = 1/3$$

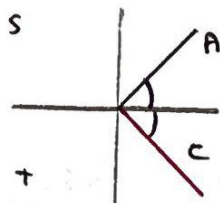
$$\cos x = 3 \times$$

since $-1 \leq \cos x \leq 1$

or $\sec x = 3$

$$\cos x = 1/3$$

P.V. $x = 1.230959$



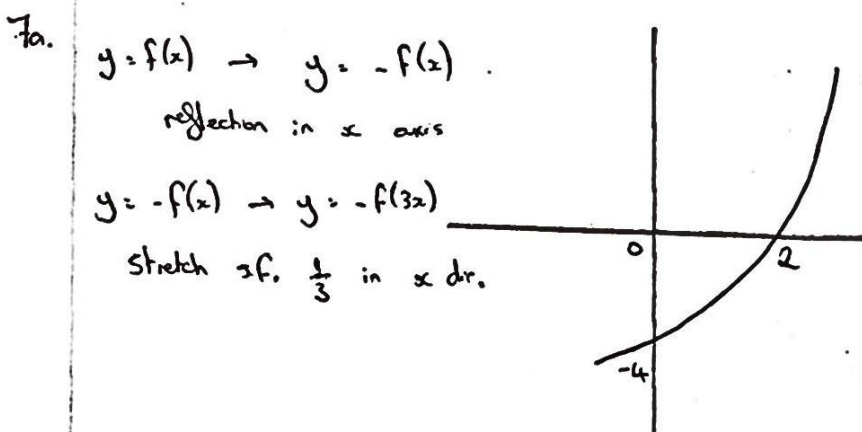
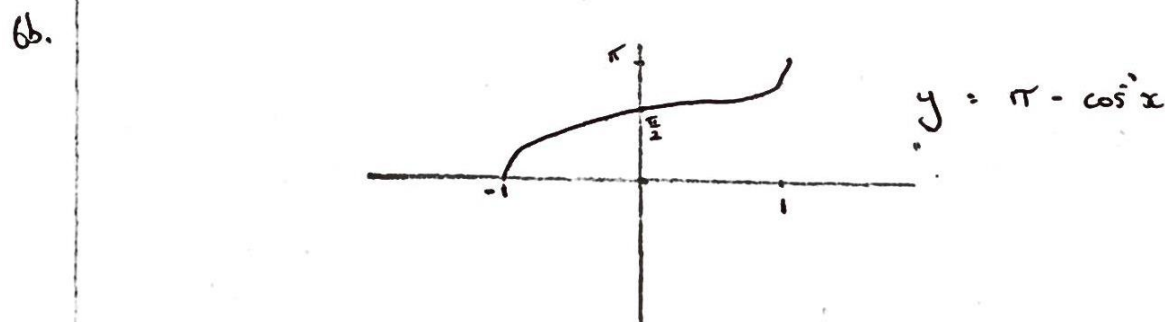
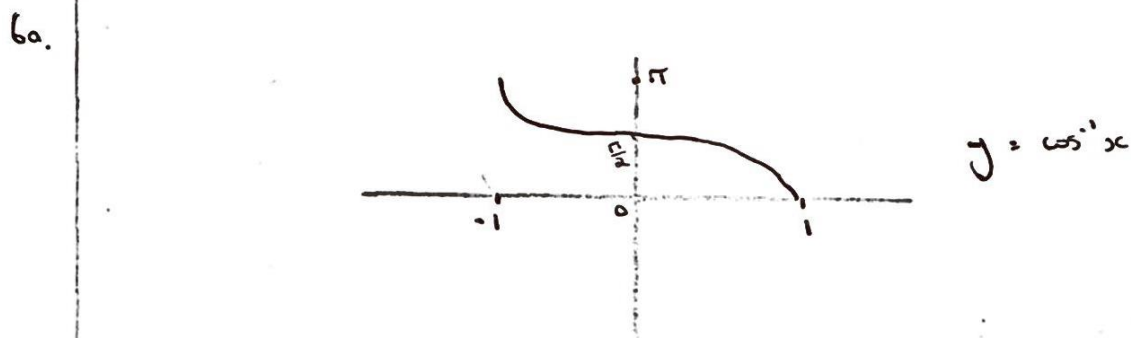
$$x = 1.23, 5.05 \quad (3 \text{ s.f.})$$

5a.

$$\int_0^4 \sqrt{27+x^3} \, dx \quad h = \frac{4-0}{5} = \frac{4}{5}$$

x $2/5$ $6/5$ 2 $14/5$ $18/5$	y $\sqrt{\frac{3383}{125}}$ $\sqrt{\frac{3591}{125}}$ $\sqrt{35}$ $\sqrt{\frac{6119}{125}}$ $\sqrt{\frac{9207}{125}}$	$\int \approx \frac{4}{5} \left\{ \sqrt{\frac{3383}{125}} + \sqrt{\frac{3591}{125}} + \sqrt{35} + \sqrt{\frac{6119}{125}} + \sqrt{\frac{9207}{125}} \right\}$ $= 25.6 \quad (3 \text{ s.f.})$
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5b. Smaller since rectangles are mostly below the curve



8b.

$$f(x) = \ln(2x-3)$$

$$g(x) = e^{2x} - 4$$

$$gf(x) = g(\ln(2x-3))$$

$$= e^{2\ln(2x-3)} - 4$$

$$= e^{\ln((2x-3)^2)} - 4$$

$$= (2x-3)^2 - 4$$

8b.

$$fg(x) = f(e^{2x} - 4)$$

$$= \ln(2e^{2x} - 8 - 3)$$

$$= \ln(2e^{2x} - 11)$$

$$\ln 5 = \ln(2e^{2x} - 11)$$

$$5 = 2e^{2x} - 11$$

$$e^{2x} = 8$$

$$2x = \ln 8$$

$$x = \frac{1}{2} \ln 8$$

9.

$$V = \int_0^{16} \pi x^2 dy$$

$$16x^2 - (y-8)^2 = 32$$

$$16x^2 = 32 + (y-8)^2$$

$$16x^2 = 32 + y^2 - 16y + 64$$

$$16x^2 = y^2 - 16y + 96$$

$$x^2 = \frac{1}{16} (y^2 - 16y + 96)$$

$$V = \frac{\pi}{16} \int_0^{16} (y^2 - 16y + 96) dy$$

$$\begin{aligned}
 &= \frac{\pi}{16} \left[\frac{1}{3}y^3 - 8y^2 + 96y \right]_0^{16} \\
 &= \frac{\pi}{16} \left\{ \left(\frac{1}{3}(16)^3 - 8(16)^2 + 96(16) - 0 \right) \right\} \\
 &= \frac{\pi}{16} \times \frac{2560}{3} \\
 &= \frac{160\pi}{3}
 \end{aligned}$$

10ax.

$$\int \ln x \, dx$$

Parts: $u = \ln x$
 $u' = \frac{1}{x}$

$v' = 1$
 $v = x$

$$= x \ln x - \int \frac{x}{x} \, dx$$

$$= x \ln x - x + c$$

10ax

$$\int (\ln x)^2 \, dx$$

Parts $u = \ln x$
 $u' = \frac{1}{x}$

$v' = \ln x$
 $v = x \ln x - x$

$$= \ln x (x \ln x - x) - \int \frac{x \ln x - x}{x} \, dx$$

$$= x \ln^2 x - x \ln x - \int \ln x - 1 \, dx$$

$$= x \ln^2 x - x \ln x - (x \ln x - x - x) + c$$

$$= x \ln^2 x - x \ln x - x \ln x + 2x + c$$

$$= x \ln^2 x - 2x \ln x + 2x + c$$

10b.

$$\int_1^4 \frac{1}{x+\sqrt{x}} dx$$

$$u = x^{1/2}$$

$$x = u^2$$

$$dx = 2u du$$

$$= \int_1^2 \frac{2u}{u^2+u} du$$

$$\frac{x}{u} \begin{array}{r} 4 \quad 1 \\ \hline 2 \quad 1 \end{array}$$

$$= 2 \int_1^2 \frac{1}{u+1} du$$

$$= 2[\ln|u+1|]_1^2$$

$$= 2 \ln 3 - 2 \ln 2$$

$$= 2 \ln\left(\frac{3}{2}\right)$$