AQA

A Level

A Level Maths

AQA Core Maths C2 June 2013 Model Solutions

Name:

M M E

Mathsmadeeasy.co.uk

Total Marks:

AQA June 13 C2

1a. GP $a = 80$ $r = \frac{1}{2}$

$u_3 = ar^2 = 80\left(\frac{1}{2}\right)^2$

$= 20$

1b. $S_\infty = \frac{a}{1-r} = \frac{80}{1-\frac{1}{2}} = 160$

1c. $S_{12} = \frac{a(1-r^{12})}{1-r} = \frac{80(1-0.5^{12})}{1-\frac{1}{2}}$

$= 159.96$ (2dp)

2a. $\ell = r\theta = 20(0.8) = 16$

2b. $A = \frac{1}{2}r^2\theta = \frac{1}{2}(20)^2(0.8) = 160$

2c. $\frac{\sin 0.8}{75} = \frac{\sin\theta}{20}$

$\theta = \sin^{-1}\left(\frac{20\sin 0.8}{75}\right)$

$= 1.2746\ldots$

$1.2746 < \frac{\pi}{2} \Rightarrow$ acute

obtuse angle: $\pi - 1.2746\ldots$

$= 1.87$ (3sf)

3ai. $(2+y)^3 = 2^3 + {}^3C_1 2^2 y + {}^3C_2 2y^2 + {}^3C_3 y^3$

$= 8 + 12y + 6y^2 + y^3$

3aii $(2+x^{-2})^3 = 8 + 12x^{-2} + 6(x^{-2})^2 + (x^{-2})^3$

$= 8 + 12x^{-2} + 6x^{-4} + x^{-6}$ ①

$(2-x^{-2})^3 = 8 - 12x^{-2} + 6(-x^{-2})^2 + (-x^{-2})^3$

$= 8 - 12x^{-2} + 6x^{-4} - x^{-6}$ ②

① + ② $= 16 + 12x^{-4}$

3bi. $\int (2+x^{-2})^3 + (2-x^{-2})^3 \, dx = \int 16 + 12x^{-4} \, dx$

$= 16x - 4x^{-3} + c$

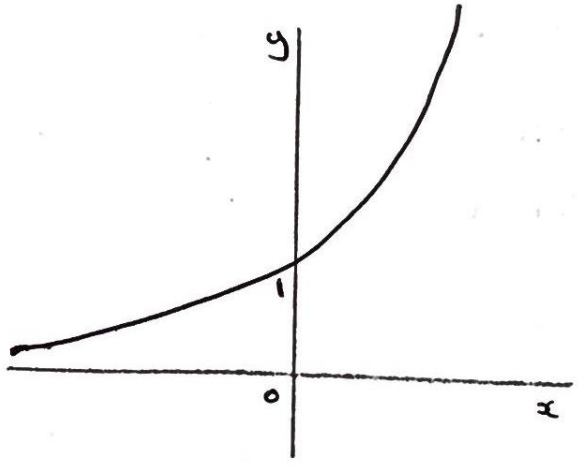
3bii $\left[16x - 4x^{-3}\right]_1^2$

$= \left(16(2) - 4(2)^{-3}\right) - \left(16(1) - 4(1)^{-3}\right)$

$= \frac{63}{2} - 12$

$= \frac{39}{2}$

4ai.



$y = 9^x$

4b. $9^x = 15$

$\log 9^x = \log 15$

$x \log 9 = \log 15$

$x = \dfrac{\log 15}{\log 9}$

$= 1.23 \quad (3sf)$

4c. $f(x) = 9^{-x}$

5a. $h = \dfrac{2-0}{4} = 1/2$

x	y
0	1
0.5	$\sqrt{2}$
1	$\sqrt{9}$
1.5	$\sqrt{28}$
2	$\sqrt{65}$

$$\int_0^2 \sqrt{8x^3+1}\,dx \approx \frac{1}{2}\cdot\frac{1}{2}\left\{(1+\sqrt{65}) + 2(\sqrt{2}+\sqrt{9}+\sqrt{28})\right\}$$

$= 7.12 \quad (3sf)$

5b. $\sqrt{8x^3+1} \longrightarrow \sqrt{x^3+1}$

$\sqrt{(2x)^3+1} \longrightarrow \sqrt{x^3+1}$ 'x replaced with $2x$'

$f(x) \rightarrow f\left(\frac{x}{2}\right)$ $\therefore$ stretch in x direction s.f. 2

5c. $y = \sqrt{x^3+1} \longrightarrow \sqrt{(x-2)^3+1}$ translation 2 right

$\sqrt{(x-2)^3+1} \longrightarrow \sqrt{(x-2)^3+1} - 0.7$ translation 0.7 down

$\therefore \quad g(4) = \sqrt{(4-2)^3+1} - 0.7$

$= 2.3$

6a. $y = \dfrac{12 + x^2\sqrt{x}}{x}$

$= \dfrac{12}{x} + x\sqrt{x}$

$= 12x^{-1} + x^{3/2}$

6bi. $\dfrac{dy}{dx} = -12x^{-2} + \dfrac{3}{2}x^{1/2}$

6bii. when $x = 4$, $\dfrac{dy}{dx} = -12(4)^{-2} + \dfrac{3}{2}(4)^{1/2}$

$= 9/4$

$\therefore$ m of $\perp$ $= -4/9$

when $x = 4$, $y = \dfrac{12 + 4^2\sqrt{4}}{4}$

$= 11$ $\quad (4, 11)$

$y - 11 = -\dfrac{4}{9}(x - 4)$

$9y - 99 = -4x + 16$

$4x + 9y = 115$

6bii. at stat point, $\dfrac{dy}{dx} = 0$

$-12x^{-2} + \dfrac{3}{2}x^{1/2} = 0$

$\dfrac{3}{2}x^{1/2} = \dfrac{12}{x^2}$

$\dfrac{3}{2}x^{5/2} = 12$

$x^{5/2} = 8$

$x = 8^{2/5}$

$x = (2^3)^{2/5} = 2^{6/5}$

7a. $u_{n+1} = pu_n + q$

$u_1 = 96, \quad u_2 = 72 \qquad l = 24$

$l = pl + q \qquad 72 = 96p + q \quad ①$

$24 = 24p + q \quad ②$

'① - ②' $\quad 48 = 72p$

$p = \frac{48}{72}$

$= \frac{2}{3}$

7b. $u_3 = pu_2 + q$

$= \frac{2}{3}(72) + 8$

$= 56$

'sub $p = 2/3$ into ②'

$24 = 24\left(\frac{2}{3}\right) + q$

$24 = 16 + q$

$q = 8$

8a. $\log_a b = c$

$b = a^c$

8b $2\log_2(x+7) - \log_2(x+5) = 3$

$\log_2(x+7)^2 - \log_2(x+5) = 3$

$\log_2\left(\frac{(x+7)^2}{x+5}\right) = 3$

$\frac{(x+7)^2}{x+5} = 2^3$

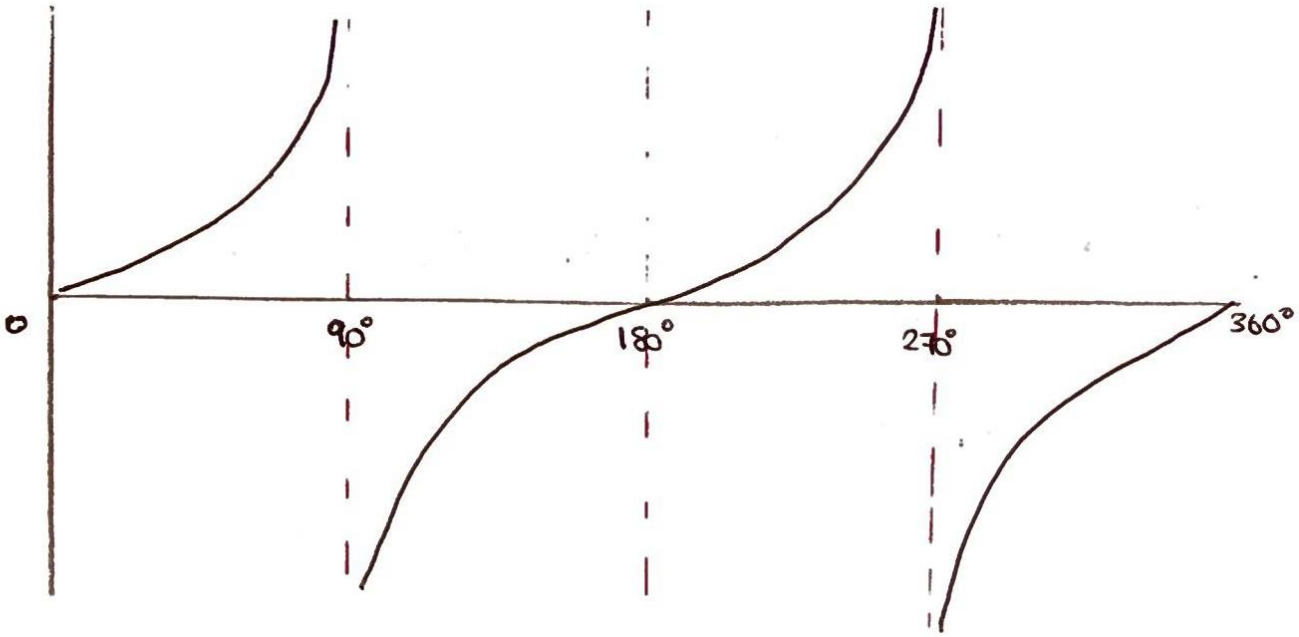
$(x+7)^2 = 8(x+5)$

$x^2 + 14x + 49 = 8x + 40$

$x^2 + 6x + 9 = 0$

$(x+3)^2 = 0 \quad \therefore x = -3$ only solution

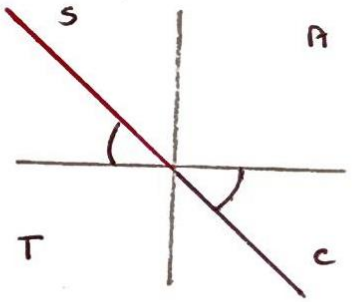
9ai.



9ai.

$\tan x = -1$ $\quad 0 \leq x \leq 360$

P.V. $x = -45°$



$x = 135°, \ 315°$

9bi.

$6 \tan\theta \sin\theta = 5$

$6 \frac{\sin\theta}{\cos\theta} \cdot \sin\theta = 5$ $\quad \frac{\sin\theta}{\cos\theta} \equiv \tan\theta$

$6 \sin^2\theta = 5\cos\theta$

$6(1 - \cos^2\theta) = 5\cos\theta$ $\quad \sin^2\theta \equiv 1 - \cos^2\theta$

$6 - 6\cos^2\theta = 5\cos\theta$

$6\cos^2\theta + 5\cos\theta - 6 = 0$

9bii.

$6\tan 3x \sin 3x = 5$

$6\cos^2 3x + 5\cos 3x - 6 = 0$

$0 \leq x \leq 180$

let $\phi = 3x$

$6\cos^2\phi + 5\cos\phi - 6 = 0$

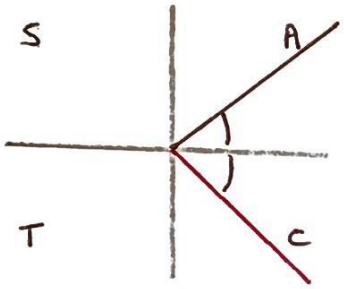
$0 \leq \phi \leq 540$

$(3\cos\phi - 2)(2\cos\phi + 3) = 0$

$\cos\phi = \frac{2}{3}$ or $\cos\phi = -3/2$ ×

$-1 \leq \cos x \leq 1 \quad \forall x \in \mathbb{R}$

P.V. $\phi = 48.19$



$\phi = 48.19, 311.81, 408.19$

$\phi = 3x$

$3x = 48.19°$ $x = 16°$ (nearest degree)

$3x = 311.81°$ $x = 104°$

$3x = 408.19°$ $x = 136°$