AQA

A Level

A Level Maths

AQA Core Maths C1 June 2013 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

AQA June 13 C1

1a. $3x - 4y + 5 = 0$ $\qquad (p, p+2)$

$x = p, \quad y = p + 2$

$3p - 4(p+2) + 5 = 0$

$3p - 4p - 8 + 5 = 0$

$p = -3$

1b. $4y = 3x + 5$

$y = \frac{3}{4}x + \frac{5}{4} \Rightarrow m = 3/4$

1c. $A(1,2) \qquad C(-5,k)$

$AC \perp$ to $AB \Rightarrow m$ of $AC = -4/3$

$\therefore \quad \frac{y - y_1}{x - x_1} = -\frac{4}{3}$

$\frac{2-k}{1--5} = -\frac{4}{3}$

$2 - k = -8$

$k = 10$

1d. $3x - 4y + 5 = 0 \quad \times 2 \quad \therefore \quad 6x - 8y + 10 = 0$ ①

$2x - 5y - 6 = 0 \quad \times 3 \quad 6x - 15y - 18 = 0$ ②

'① - ②' $\quad 7y + 28 = 0 \Rightarrow y = -4$

'sub $y = -4$ into ①' $\quad 3x - 4(-4) + 5 = 0$

$3x + 21 = 0$

$x = -7$

2ai. $\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

2aii. $x\sqrt{12} = 7\sqrt{3} - \sqrt{48}$

$x\sqrt{12} = 3\sqrt{3}$ $\quad (\sqrt{48} = 4\sqrt{3})$

$x\sqrt{4 \times 3} = 3\sqrt{3}$

$2x\sqrt{3} = 3\sqrt{3}$

$2x = 3 \Rightarrow x = 3/2$

2b.

$$\frac{11\sqrt{3} + 2\sqrt{5}}{2\sqrt{3} + \sqrt{5}} \times (2\sqrt{3} - \sqrt{5})$$

$$\frac{(11\sqrt{3} + 2\sqrt{5})(2\sqrt{3} - \sqrt{5})}{(2\sqrt{3} + \sqrt{5})(2\sqrt{3} - \sqrt{5})} = \frac{22(2) - 11\sqrt{15} + 4\sqrt{15} - 10}{12 - 5}$$

$$= \frac{56 - 7\sqrt{15}}{7}$$

$$= 8 - \sqrt{15}$$

3a.

$x^2 + y^2 - 10x + 14y + 25 = 0$

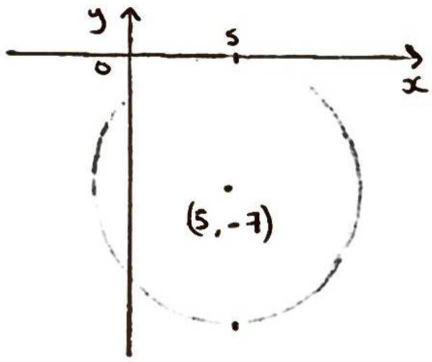
$(x-5)^2 - 25 + (y+7)^2 - 49 + 25 = 0$

$(x-5)^2 + (y+7)^2 = 49$

3bi. $(5, -7)$

3bii. 7

3ci.



3cii. $(5, -14)$ (7 units down from centre)

3d. Centre of new circle : $(-1, 0)$

$\therefore$ translation 6 right, 7 down

4ai. $f(x) = x^3 - 4x + 15$

$f(-3) = (-3)^3 - 4(-3) + 15$

$= -27 + 12 + 5$

$= 0 \quad \therefore \ (x+3)$ is a factor

4aii.

$$\begin{array}{r|l} & x^2 - 3x + 5 \\ x+3 & x^3 + 0x^2 - 4x + 15 \\ & x^3 + 3x^2 \\ \hline & -3x^2 - 4x \\ & -3x^2 - 9x \\ \hline & 5x + 15 \\ & 5x + 15 \\ \hline & 0 \end{array}$$

$f(x) = (x+3)(x^2 - 3x + 5)$

4bi. $y = x^4 - 8x^2 + 60x + 7$

$\frac{dy}{dx} = 4x^3 - 16x + 60$

4bii. $\frac{dy}{dx} = 0$ at stat points

$4x^3 - 16x + 60 = 0 \quad (\div 4)$

$x^3 - 4x + 15 = 0$

4biii. $(x+3)(x^2 - 3x + 5) = 0$

$x = -3$ or $x^2 - 3x + 5 = 0$

disc. $(-3)^2 - 4(1)(5)$

$= 9 - 20$

$= -11$

disc < 0 $\therefore$ no real roots

$\therefore$ $x = -3$ only stat. point

4biv. $\frac{d^2y}{dx^2} = 12x^2 - 16$

when $x = -3$, $\frac{d^2y}{dx^2} = 12(-3)^2 - 16$

$= 108 - 16$

$= 92$

4bv. $92 > 0 \Rightarrow$ minimum at $x = -3$

5ai. $2x^2 + 6x + 5 = 2\left[x^2 + 3x + 5/2\right]$

$= 2\left[(x + 3/2)^2 - 9/4 + 5/2\right]$

$= 2(x + 3/2)^2 - 9/2 + 5$

$= 2(x + 3/2)^2 + 1/2$

5aii. $1/2$

5bi. A $(-3, 5)$ B $(x, 3x + 9)$

$AB^2 = (x - -3)^2 + (3x + 9 - 5)^2$

$= (x + 3)^2 + (3x + 4)^2$

$= x^2 + 6x + 9 + 9x^2 + 24x + 16$

$= 10x^2 + 30x + 25$

$= \cancel{10}\ 5(2x^2 + 6x + 5)$

5bii. min value of $2x^2 + 6x + 5 = \frac{1}{2}$

so min $AB^2 = 5 \times \frac{1}{2}$

$AB = \sqrt{\frac{5}{2}}$

$= \frac{\sqrt{5}}{\sqrt{2}}$

$= \frac{\sqrt{10}}{2}$

$= \frac{1}{2}\sqrt{10}$

6a. $y = x^5 - 2x^2 + 9$ $\qquad P(-1, 6)$

$\frac{dy}{dx} = 5x^4 - 4x$

at P, $\frac{dy}{dx} = 5(-1)^4 - 4(-1)$

$= 9$

$\therefore$ m of tangent $= 9$

$y - 6 = 9(x - -1)$

$y - 6 = 9x + 9$

$y = 9x + 15$

6bi. when $x = 2$, $y = (2)^5 - 2(2)^2 + 9$

$= 32 - 8 + 9$

$= 33$

$\therefore k = 33$

6bii. $33 = 9(2) + 15$

$= 18 + 15$

$33 = 33$ ✓ $\therefore$ Q lies on tangent

6ci. $\int_{-1}^{2} x^5 - 2x^2 + 9 \, dx$

$= \left[\frac{1}{6}x^6 - \frac{2}{3}x^3 + 9x\right]_{-1}^{2}$

$= \left(\frac{1}{6}(2)^6 - \frac{2}{3}(2)^3 + 9(2)\right) - \left(\frac{1}{6}(-1)^6 - \frac{2}{3}(-1)^3 + 9(-1)\right)$

$= \left(\frac{64}{6} - \frac{16}{3} + 18\right) - \left(\frac{1}{6} + \frac{2}{3} - 9\right)$

$= \left(\frac{64}{6} - \frac{32}{6} + \frac{108}{6}\right) - \left(\frac{1}{6} + \frac{4}{6} - \frac{54}{6}\right) = \frac{189}{6}$

6cii. A of trapezium $= \frac{1}{2}(6+33) \times 3$

$= \frac{117}{2}$

Shaded $= \frac{117}{2} - \frac{189}{6}$

$= \frac{351}{6} - \frac{189}{6}$

$= \frac{162}{6}$

$= 27$

7a. $(2k-7)x^2 - (k-2)x + (k-3) = 0$

real roots $\Leftrightarrow$ $b^2 - 4ac \geqslant 0$

$(k-2)^2 - 4(2k-7)(k-3) \geqslant 0$

$k^2 - 4k + 4 - 4(2k^2 - 13k + 21) \geqslant 0$

$k^2 - 4k + 4 - 8k^2 + 52k - 84 \geqslant 0$

$7k^2 - 48k + 80 \leq 0$

7b. $(7k-20)(k-4) \leq 0$

c.v. $k = 4$ $k = \frac{20}{7}$

$\therefore \frac{20}{7} \leq k \leq 4$