AQA

A Level

A Level Maths

AQA Core Maths C4 June 2012 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

AQA June 12 C4

1ai.

$$\frac{5x-6}{x(x-3)} = \frac{A}{x} + \frac{B}{x-3}$$

$$5x-6 = A(x-3) + Bx$$

$x=0$; $\quad -6 = -3A \quad \Rightarrow \quad A = 2$

$x=3$; $\quad 9 = 3B \quad \Rightarrow \quad B = 3$

$$\frac{5x-6}{x(x-3)} = \frac{2}{x} + \frac{3}{x-3}$$

1aii.

$$\int \frac{2}{x} + \frac{3}{x-3} \, dx$$

$$= 2\ln x + 3\ln|x-3| + c$$

1bi.

$$\begin{array}{r l}
 & 2x^2 - x + 3 \\
2x+1 \,\big) & 4x^3 + 0x^2 + 5x - 2 \\
 & \underline{4x^3 + 2x^2} \quad \downarrow \quad \downarrow \\
 & -2x^2 + 5x \\
 & \underline{-2x^2 - x} \\
 & 6x - 2 \\
 & \underline{6x + 3} \\
 & -5
\end{array}$$

$$4x^3 + 5x - 2 = (2x+1)(2x^2 - x + 3) - 5$$

1bii.

$$\int 2x^2 - x + 3 - \frac{5}{2x+1} \, dx$$

$$= \frac{2}{3}x^3 - \frac{1}{2}x^2 + 3x - \frac{5}{2}\ln|2x+1| + c$$

2a. $\sin x - 3\cos x = R\sin(x - \alpha)$

$\sin x - 3\cos x = R\sin x\cos\alpha - R\cos x\sin\alpha$

$R = \sqrt{1^2+3^2} = \sqrt{10}$

$\sin x$: $1 = \sqrt{10}\cos\alpha$

$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$

$\alpha = 71.6°$ (1dp)

$\sin x - 3\cos x = \sqrt{10}\sin(x - 71.6)$

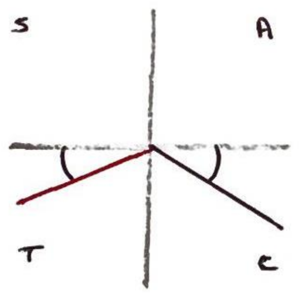
2b. $\sqrt{10}\sin(x - 71.6) = -2$ $\quad 0 < x < 360$

$\sin(x - 71.6) = -\frac{2}{\sqrt{10}}$

let $\phi = x - 71.6$ $\quad -71.6 < \phi < 288.4$

$\sin\phi = -\frac{2}{\sqrt{10}}$

P.V. $\phi = -39.2°$



$\phi = -39.2°, \ 219.9°$

$x = 32.4°, \ 290.8°$

3a. $(1+4x)^{1/2} \simeq 1 + \left(\frac{1}{2}\right)(4x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(4x)^2 + \ldots$

$= 1 + 2x - 2x^2 + \ldots$

3bi. $(4-x)^{-1/2} = \left[4\left(1 - x/4\right)\right]^{-1/2}$

$= 4^{-1/2}\left(1 - x/4\right)^{-1/2}$

$= \frac{1}{2}\left(1 - x/4\right)^{-1/2}$

$= \frac{1}{2}\left(1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)^2 + \ldots\right)$

$= \frac{1}{2}\left(1 + \frac{x}{8} + \frac{3}{128}x^2 + \ldots\right.$

$= \frac{1}{2} + \frac{x}{16} + \frac{3x^2}{256} + \ldots$

3bii. $\left|\frac{x}{4}\right| < 1 \Rightarrow |x| < 4$

3c. $\sqrt{\frac{1+4x}{4-x}} = (1+4x)^{1/2}(4-x)^{-1/2}$

$= \left(1 + 2x - 2x^2 + \ldots\right)\left(\frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2 + \ldots\right)$

$= \frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2 + x + \frac{1}{8}x^2 + \ldots - x^2 - \frac{1}{8}x^2 + \ldots$

$= \frac{1}{2} + \frac{17}{16}x - \frac{221}{256}x^2 + \ldots$

4ai. $V = P\left(1 + \frac{r}{100}\right)^n$

$r = 3, \quad P = 1000, \quad n = 5$

$V = 1000\left(1 + \frac{3}{100}\right)^5$

$= £1159.27$

$= £1160$ (nearest £10)

4aii. $V = 2.000$

$2000 = 1000\left(1 + \frac{3}{100}\right)^N$

$2 = 1.03^N$

$\ln 2 = N \ln 1.03$

$N = \frac{\ln 2}{\ln 1.03}$

$= 23.45\ldots$

so after 24 years

4b

Mr. B. $V_1 = 1000(1.03)^n$

Mrs. W. $V_2 = 1500 \times 1.015^n$

$1000(1.03)^T = 1500(1.015)^T$

$1.03^T = \frac{3}{2}(1.015^T)$

$\left(\frac{1.03}{1.015}\right)^T = \frac{3}{2}$

$T \ln\left(\frac{1.03}{1.015}\right) = \ln(3/2)$

$T = \frac{\ln(3/2)}{\ln\left(\frac{1.03}{1.015}\right)} = 27.64$

$\therefore T = 28$ years

5ai.

$x = 2\cos\theta \qquad y = 3\sin 2\theta$

$\frac{dx}{d\theta} = -2\sin\theta \qquad \frac{dy}{d\theta} = 6\cos 2\theta$

$$\frac{dy}{dx} = \frac{6\cos 2\theta}{-2\sin\theta}$$

$$= \frac{-3(1-2\sin^2\theta)}{\sin\theta} \qquad (\cos 2\theta \equiv 1 - 2\sin^2\theta)$$

$$= \frac{-3}{\sin\theta} + \frac{6\sin^2\theta}{\sin\theta}$$

$$= 6\sin\theta - 3\operatorname{cosec}\theta$$

5aii.

when $\theta = \frac{\pi}{6}$, $\frac{dy}{dx} = 6\sin(\pi/6) - \frac{3}{\sin(\pi/6)}$

$$= -3$$

$\therefore$ grad of normal $= \frac{1}{3}$ (since $\perp$)

5b.

$x = 2\cos\theta$

$\cos\theta = \frac{x}{2}$

$\cos^2\theta = \frac{x^2}{4}$

$\cos^4\theta = \frac{x^4}{16}$

$y = 3\sin 2\theta$

$y = 6\sin\theta\cos\theta$

$y^2 = 36\sin^2\theta\cos^2\theta$

$= 36\cos^2\theta(1 - \cos^2\theta)$

$= 36\cos^2\theta - 36\cos^4\theta$

$$y^2 = 36.\frac{x^2}{4} - 36.\frac{x^4}{16}$$

$$= 9x^2 - \frac{9}{4}x^4$$

$$= \frac{9}{4}x^2(4 - x^2)$$

6. $9x^2 - 6xy + 4y^2 = 3$

$$18x - 6y - 6x\frac{dy}{dx} + 8y\frac{dy}{dx} = 0$$

$$18x - 6y = \frac{dy}{dx}(6x - 8y)$$

$$\frac{dy}{dx} = \frac{18 - 6y}{6x - 8y}$$

at stat pt $\frac{dy}{dx} = 0$

$$\frac{18x - 6y}{6x - 8y} = 0$$

$$18x - 6y = 0$$

$$18x = 6y$$

$$y = 3x$$ - sub into curve

$$9x^2 - 6x(3x) + 4(3x)^2 = 3$$

$$9x^2 - 18x^2 + 36x^2 = 3$$

$$27x^2 = 3$$

$$x^2 = \frac{1}{9}$$

$$x = \pm\frac{1}{3}$$

when $x = \frac{1}{3}$ $\quad y = 3(\frac{1}{3}) = 1 \quad (1/3, 1)$

$x = -\frac{1}{3}$ $\quad y = 3(-\frac{1}{3}) = -1 \quad (1/3, -1)$

7ai. $l_1 : \underline{r} = \begin{pmatrix} 0 \\ -2 \\ q \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$

$l_2 = \underline{r} = \begin{pmatrix} 8 \\ 3 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix}$

at P :

$2\lambda = 8 + 2\mu$ $\quad \underline{i}$

$-2 = 3 + 5\mu$ $\quad \underline{j}$

$q - \lambda = 5 + 4\mu$ $\quad \underline{k}$

$\underline{j}$ $\quad -2 = 3 + 5\mu$

$-5 = 5\mu \Rightarrow \mu = -1$

'sub $\mu = -1$ into $\underline{i}$' $\quad 2\lambda = 8 - 2$

$\lambda = 3$

'sub $\mu = -1$, $\lambda = 3$ into $\underline{k}$' $\quad q - 3 = 5 - 4$

$q - 3 = 1$

$q = 4$

sub $\lambda = 3$ into l_1 $\quad P\ (6, -2, 1)$

7b. $\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix} = 2(2) + 0 - 1(4)$

$= 4 - 4$

$= 0 \quad \therefore \underline{\perp}$

7ci. at A, $\lambda = 1$: $\begin{pmatrix} 2(1) \\ -2 \\ 4-1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix} = \overrightarrow{OA}$

$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA}$

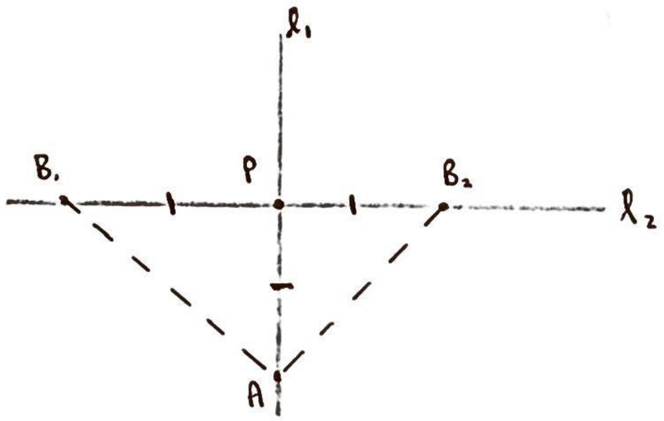
$= \begin{pmatrix} 6 \\ -2 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$

$AP^2 = 4^2 + 0^2 + (-2)^2$

$= 20$

7b.



Isosceles $\therefore$ $|AP| = |PB| = \sqrt{20}$

B lies on l_2 so $\overrightarrow{OB} = \begin{pmatrix} 8+2m \\ 3+5m \\ 5+4m \end{pmatrix}$ for some m

$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP}$

$= \begin{pmatrix} 8+2m \\ 3+5m \\ 5+4m \end{pmatrix} - \begin{pmatrix} 6 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2+2m \\ 5+5m \\ 4+4m \end{pmatrix}$

$|PB| = \sqrt{20} = \sqrt{(2+2m)^2 + (5+5m)^2 + (4+4m)^2}$

$20 = (2+2m)^2 + (5+5m)^2 + (4+4m)^2$

$20 = 4 + 8m + 4m^2 + 25 + 50m + 25m^2 + 16 + 32m + 16m^2$

$45m^2 + 90m + 25 = 0$

$9m^2 + 18m + 5 = 0$

$(3m+5)(3m+1) = 0$

$m = -\frac{5}{3}$ or $m = -\frac{1}{3}$

when $m = -\frac{5}{3}$ $\quad \vec{OB} = \begin{pmatrix} 14/3 \\ -14/3 \\ -5/3 \end{pmatrix}$ $\quad B\left(\frac{14}{3}, -\frac{16}{3}, -\frac{5}{3}\right)$.

$m = -\frac{1}{3}$ $\quad \vec{OB} = \begin{pmatrix} 22/3 \\ 4/3 \\ 11/3 \end{pmatrix}$ $\quad B\left(\frac{22}{3}, \frac{4}{3}, \frac{11}{3}\right)$.

8a

$\frac{dh}{dt} \propto \frac{\cancel{2(-h}}{2-h}$ $\quad$ so $\frac{dh}{dt} = k(2-h)$

8bi.

$\frac{dx}{dt} = \frac{1}{15x\sqrt{2x-1}}$

$\int \cancel{15}x\sqrt{2x-1}\;dx = \int \frac{1}{15}\,dt$

$\int \cancel{15}x\sqrt{2x-1}\;dx$

let $u = 2x-1$

$x = \frac{1}{2}(u+1)$

$dx = \frac{1}{2}\,du$

$\int \frac{1}{2}(u+1).\,u^{1/2}.\,\frac{1}{2}\,du$

$= \frac{1}{4}\int u^{1/2}(u+1)\,du$

$= \frac{1}{4}\int u^{3/2} + u^{1/2}\,du$

$= \frac{1}{4}\left(\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right) + c$

$\frac{1}{4}\left(\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right) = \frac{1}{15}t + c$

when $t=0$, $x=1$. $\quad$ when $x=1$, $\quad u = 2(1)-1$
$= 1$

$\frac{1}{4}\left(\frac{2}{5} + \frac{2}{3}\right) = 0 + c$ $\qquad c = \frac{4}{15}$

$$\frac{1}{4}\left(\frac{2}{5}(2x-1)^{5/2}+\frac{2}{3}(2x-1)^{3/2}\right)=\frac{1}{15}t+\frac{4}{15}$$

$$\frac{1}{15}t=\frac{1}{4}\left(\frac{2}{5}(2x-1)^{5/2}+\frac{2}{3}(2x-1)^{3/2}\right)-\frac{4}{15}$$

$$t=\frac{15}{4}\left(\frac{2}{5}(2x-1)^{5/2}+\frac{2}{3}(2x-1)^{3/2}\right)-4$$

$$t=\frac{3}{2}(2x-1)^{5/2}+\frac{5}{2}(2x-1)^{3/2}-4$$

8bii when $x=2$, $t=\frac{3}{2}(4-1)^{5/2}+\frac{5}{2}(4-1)^{3/2}-4$

$$=32.4 \quad (1\text{ dp})$$