AQA

A Level

A Level Maths

AQA Core Maths C3 June 2012

Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

AQA June 12 C3

1. $\int_{0.4}^{1.2} \cot(x^2)\ dx$ $\quad h = \frac{1.2 - 0.4}{4} = 0.2$

x	y
0.5	$\cot(0.25)$
0.7	$\cot(0.49)$
0.9	$\cot(0.81)$
1.1	$\cot(1.21)$

$$\int \approx 0.2\left\{\cot(0.25) + \cot(0.49) + \cot(0.81) + \cot(1.21)\right\}$$

$$= 1.424 \quad (3\text{dp})$$

2a. $y = 4\ln x, \quad y = \sqrt{x}$

for intersection $4\ln x = \sqrt{x}$

let $f(x) = 4\ln x - \sqrt{x}$

$f(1.5) = 0.397115\ldots$

$f(0.5) = -3.47969\ldots$

change of sign $\Rightarrow 0.5 < \alpha < 1.5$

2b. $4\ln x = \sqrt{x}$

$\ln x = \frac{\sqrt{x}}{4}$

$x = e^{\frac{\sqrt{x}}{4}}$

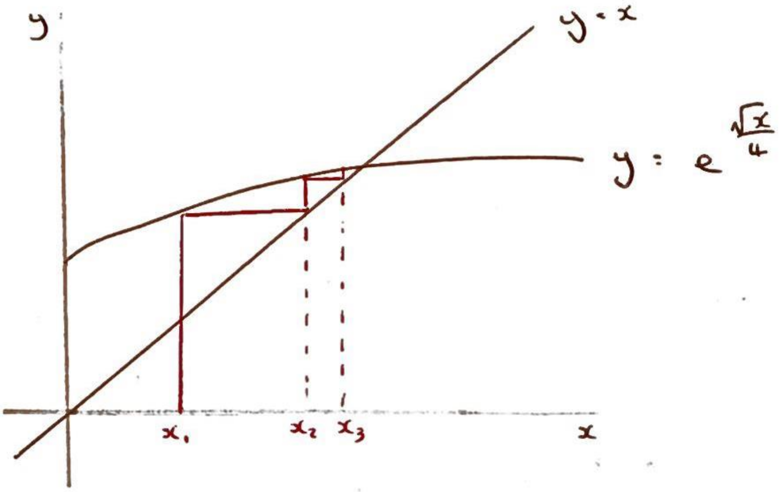
2c. $x_{n+1} = e^{\frac{\sqrt{x_n}}{4}}$

$x_1 = 0.5$

$x_2 = 1.193$

$x_3 = 1.314$

2d.



3a.

$$y = x^3 \ln x$$

$$\frac{dy}{dx} = 3x^2 \ln x + x^3 \cdot \frac{1}{x}$$

$$= 3x^2 \ln x + x^2$$

3bi.

when $x = e$, $y = e^3 \ln e$

$$= e^3 \qquad (e, e^3)$$

when $x = e$, $\frac{dy}{dx} = 3e^2 \ln e + e^2$

$$= 4e^2$$

$$y - e^3 = 4e^2(x - e)$$

3bii.

when curve intersects the x axis, $y = 0$

$$0 - e^3 = 4e^2(x - e)$$

$$4e^2x - 4e^3 + e^3 = 0$$

$$4e^2x = 3e^3 \qquad (\text{since } e^2 > 0 \text{ can divide by } e^2)$$

$$4x = 3e$$

$$x = \frac{3}{4}e$$

$$\therefore \; A \left(\tfrac{3}{4}e, 0\right)$$

5ci. Let $y = \sqrt{2x-5}$

$$y^2 = 2x - 5$$

$$y^2 + 5 = 2x$$

$$x = \frac{1}{2}(y^2+5)$$

$$\therefore \quad f^{-1}(x) = \frac{1}{2}(x^2+5)$$

5cii. $f^{-1}(x) = 7 \quad \Rightarrow \quad \frac{1}{2}(x^2+5) = 7$

$$x^2 + 5 = 14$$

$$x^2 = 9$$

$$x = \pm 3$$

domain of $f^{-1}(x)$ = range of $f(x)$ = $f(x) \geq 0$

$\therefore$ domain $f^{-1}(x)$; $x \geq 0$

$$\therefore \quad x = 3$$

6. $$\int_0^1 \frac{x^7}{(x^4+2)^2} \, dx$$

$u = x^4 + 2$

$x = (u-2)^{1/4}$

$dx = \frac{1}{4}(u-2)^{-3/4} \, du$

$x^7 = (u-2)^{7/4}$

x	1	0
u	3	2

$$= \int_2^3 \frac{(u-2)^{7/4}}{u^2} \cdot \frac{1}{4}(u-2)^{-3/4} \, du$$

$$= \frac{1}{4}\int_2^3 \frac{u-2}{u^2} \, du$$

$$= \frac{1}{4}\int_2^3 \frac{1}{u} - 2u^{-2} \, du$$

$$= \frac{1}{4}\left[\ln u + \frac{2}{u}\right]_2^3$$

$$= \frac{1}{4}\left(\left(\ln 3 + \frac{2}{3}\right) - (\ln 2 + 1)\right)$$

$$= \frac{1}{4}\ln\frac{3}{2} - \frac{1}{12}$$

4a. $\int xe^{6x}\,dx$ Parts: $u = x$ $v' = e^{6x}$

$u' = 1$ $v = \frac{1}{6}e^{6x}$

$= x.\frac{1}{6}e^{6x} - \int 1.\frac{1}{6}e^{6x}\,dx$

$= \frac{x}{6}e^{6x} - \frac{1}{36}e^{6x} + c$

4b. $V = \pi\int_0^1 y^2\,dx$ $y = \sqrt{x}\,e^{3x}$

$= \pi\int_0^1 xe^{6x}\,dx$ $y^2 = xe^{6x}$

$= \pi\left[\frac{x}{6}e^{6x} - \frac{1}{36}e^{6x}\right]_0^1$

$= \pi\left\{\left(\frac{1}{6}e^6 - \frac{1}{36}e^6\right) - \left(-\frac{1}{36}\right)\right\}$

$= \pi\left\{\frac{5}{36}e^6 + \frac{1}{36}\right\}$

$\therefore p = \frac{5}{36}, \quad q = \frac{1}{36}$

5a. $f(x) = \sqrt{2x-5}$ $x \geq 2.5$

$g(x) = \frac{10}{x}$ $x \neq 0$

$f(2.5) = \sqrt{5-5} = 0$

$\therefore f(x) \geq 0$

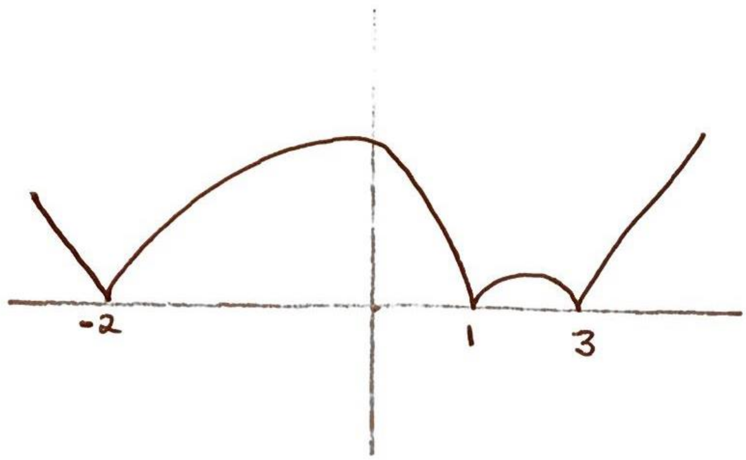
5b. $fg(x) = f\left(\frac{10}{x}\right) = \sqrt{\frac{20}{x} - 5}$

5bi. $fg(x) = 5 \Rightarrow \sqrt{\frac{20}{x} - 5} = 5$

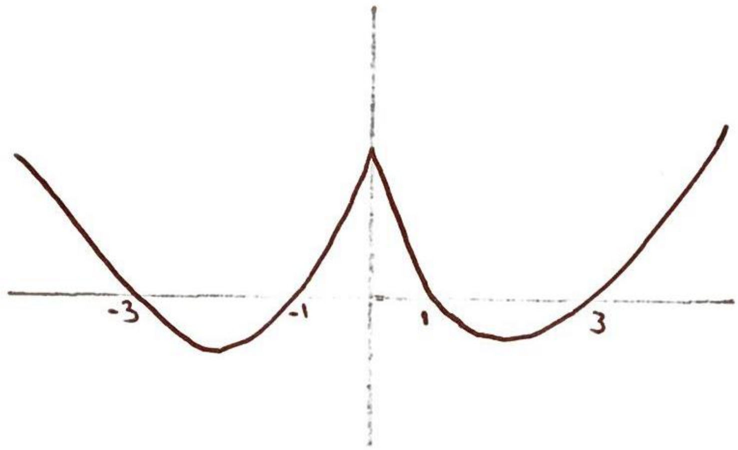
$\frac{20}{x} - 5 = 25$

$\frac{20}{x} = 30 \Rightarrow x = 2/3$

7a.



7b.



7c.

$y = f(x) \rightarrow y = \frac{1}{2} f(x)$ stretch s.f. $\frac{1}{2}$ in y direction

$y = \frac{1}{2} f(x) \rightarrow y = \frac{1}{2} f(x+1)$ translation $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

7d.

$(-1, 10)$ stretch sf. $\frac{1}{2}$ in y => y values halved

translation $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ so x values decrease by 1

$(-1, 10) \rightarrow (-2, 5)$

8a.

$$\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} = 32$$

$$\frac{1-\cos\theta + 1+\cos\theta}{(1+\cos\theta)(1-\cos\theta)} = 32$$

$$\frac{2}{1-\cos^2\theta} = 32$$

$$\frac{1}{1-\cos^2\theta} = 16$$

$$\frac{1}{\sin^2\theta} = 16 \qquad (\sin^2\theta \equiv 1-\cos^2\theta)$$

$$\operatorname{cosec}^2\theta = 16$$

8b.

$$\frac{1}{1+\cos(2x-0.6)} + \frac{1}{1-\cos(2x-0.6)} = 32 \qquad 0 < x < \pi$$

$$\operatorname{cosec}^2(2x-0.6) = 16$$

$$\operatorname{cosec}(2x-0.6) = \pm 4$$

$$\sin(2x-0.6) = \pm \frac{1}{4}$$

let $\phi = 2x - 0.6$

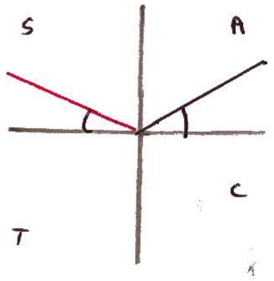
$0 < x < \pi$

$0 < 2x < 2\pi$

$-0.6 < \phi < 2\pi - 0.6$

$\sin\phi = 1/4$

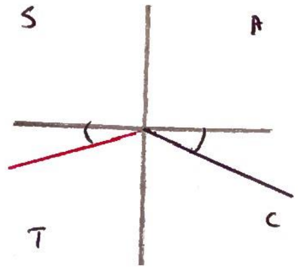
P.V. $\phi = 0.25268$



$\phi = 0.25268, \ 2.88891,$

or $\sin\phi = -\frac{1}{4}$

P.V. $\phi = -0.25268$



$\phi = -0.25268, \ 3.3943$

$\phi = 2x - 0.6$

$\phi = 0.25268 \Rightarrow x = 0.43$

$= 2.88891 \Rightarrow x = 1.74$

$= -0.25268 \Rightarrow x = 0.17$

$= 3.3943 \Rightarrow x = 2.00$ (2d.p.)

9a.

$x = \dfrac{\sin y}{\cos y}$

$f = \sin y \qquad g = \cos y$

$f' = \cos y \qquad g' = -\sin y$

$\dfrac{dx}{dy} = \dfrac{\cos^2 y - (-\sin^2 y)}{\cos^2 y}$

$= \dfrac{1}{\cos^2 y} = \sec^2 y$

9b.

$\tan y = x - 1$

$\tan^2 y = (x-1)^2$

$\tan^2 y + 1 = (x-1)^2 + 1$

$\sec^2 y = (x-1)^2 + 1 \qquad (\sec^2 y \equiv \tan^2 y + 1)$

$= x^2 - 2x + 2$

9c.

$\dfrac{dx}{dy} = \sec^2 y \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sec^2 y} = \dfrac{1}{x^2 - 2x + 2}$

9di.

$y = \tan^{-1}(x-1) - \ln x$

$\dfrac{dy}{dx} = \dfrac{1}{x^2 - 2x + 2} - \dfrac{1}{x}$

at stat. points. $\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{1}{x} = \dfrac{1}{x^2 - 2x + 2}$

$\therefore x = x^2 - 2x + 2$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 2$ or $x = 1$

9dii.

$\frac{dy}{dx} = \frac{1}{x^2-2x+2} - \frac{1}{x}$ Quotient : $f = 1$ $g = x^2 - 2x + 2$

$f' = 0$ $g' = 2x - 2$

$\frac{d^2y}{dx^2} = \frac{0 - 1(2x-2)}{(x^2-2x+2)^2} + x^{-2}$ $\left(\frac{d}{dx} - x^{-1} = x^{-2} \right)$

$= \frac{2-2x}{(x^2-2x+2)^2} + \frac{1}{x^2}$

9diii.

when $x = 1$, $y = \tan^{-1}(1-1) - \ln 1$

$= 0$ stat. point at $(1, 0)$

when $x = 2$, $y = \tan^{-1}(2-1) - \ln 2$

$= 0.09225 \ldots$ st. pt. at $(2, 0.09\ldots)$

so $(1,0)$ only stat point on x axis

when $x = 1$, $\frac{d^2y}{dx^2} = \frac{2-2(1)}{(1^2-2(1)+2)^2} + \frac{1}{1^2}$

$= 1$

$\frac{d^2y}{dx^2} > 0$

$\therefore$ minimum point at $(1, 0)$