AQA

A Level

A Level Maths

AQA Core Maths C2 June 2012
Model Solutions

Name:

MME
Mathsmadeeasy.co.uk

Total Marks:

AQA June 12 C2

1a. $23 + 32 + 41 + 50 + \dots + 2534$

$d = 9$, $a = 23$

1b. $u_{100} = a + 99d$

$= 23 + 99(9)$

$= 914$

1c. $S_{280} = \frac{1}{2}(280)(a + l)$

$= 140\ (23 + 2534)$

$= 357,980$

2a. $A = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(31.5)(26) \times \frac{5}{13}$

$= 157.5$

2b. $\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = \sqrt{1 - \sin^2\theta}$

$= \sqrt{1 - (5/13)^2}$ $\quad (\sin\theta = 5/13)$

$= \frac{12}{13}$

2c. $l^2 = a^2 + b^2 - 2ab\cos C$

$= 26^2 + 31.5^2 - 2(26)(31.5) \cdot \frac{12}{13}$

$l = \sqrt{156.25}$

$= 12.5$

3a. $\left(x^{3/2}-1\right)^2 = x^{6/4 \cdot 2} - x^{3/2} - x^{3/2} + 1$

$= x^3 - 2x^{3/2} + 1$

3b. $\int x^3 - 2x^{3/2} + 1 \; dx$

$= \frac{1}{4}x^4 - \frac{4}{5}x^{5/2} + x + c$

3c. $\left[\frac{1}{4}x^4 - \frac{4}{5}x^{5/2} + x\right]_1^4$

$= \left(\frac{1}{4}(4)^4 - \frac{4}{5}(4)^{5/2} + 4\right) - \left(\frac{1}{4}(1)^4 - \frac{4}{5}(1)^{5/2} + 1\right)$

$= \frac{212}{5} - \frac{9}{20}$

$= \frac{839}{20}$

4a. $U_n = 48\left(\frac{1}{4}\right)^n$

$U_1 = 48\left(\frac{1}{4}\right)^1 = 12$

$U_2 = 48\left(\frac{1}{4}\right)^2 = 3$

4b. $r = \frac{U_2}{U_1} = \frac{1}{4}$

4c. $S_\infty = \frac{a}{1-r} = \frac{12}{1-1/4} = 16$

4d. $\sum_{n=4}^{\infty} U_n = U_4 + U_5 + \dots + U_\infty$

$= S_\infty - S_3$

$U_3 = 48\left(\frac{1}{4}\right)^3 = 3/4$, $\quad S_3 = 12 + 3 + 3/4 = 63/4$

$\sum_{n=4}^{\infty} U_n = 16 - 63/4 = \frac{1}{4}$

5a. $l = r\theta$

$= 18 \times \frac{2\pi}{3}$

$= 12\pi$

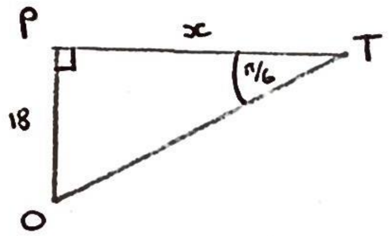
5bi. $\alpha + \frac{\pi}{2} + \frac{\pi}{2} + \frac{2\pi}{3} = 2\pi$ (angles in a quadrilateral sum to 2π)

$\alpha = \frac{1}{3}\pi$

5bii. Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}(18)^2\left(\frac{2\pi}{3}\right)$

$= 108\pi$



$\tan\left(\frac{\pi}{6}\right) = \frac{18}{x}$

$x = \frac{18}{\tan(\pi/6)} = 18\sqrt{3}$

Area of $\triangle = \frac{1}{2}\, 18\sqrt{3} \times 18 = 162\sqrt{3}$

$\Rightarrow$ Area of kite PTQO $= 162\sqrt{3} \times 2$

$= 324\sqrt{3}$

Shaded Area = kite - sector

$= 324\sqrt{3} - 108\pi$

$= 222$ (3sf)

6ai. $\frac{dy}{dx} = 3x^2 - \frac{4}{x^2} - 11$

when $x = 2$, $\frac{dy}{dx} = 3(2)^2 - \frac{4}{2^2} - 11$

$= 12 - 1 - 11$

$= 0$

6aii. $\frac{d^2y}{dx^2} = 6x + 8x^{-3}$

when $x = 2$, $\frac{d^2y}{dx^2} = 6(2) + 8(2)^{-3}$

$= 13$

6aiii. $\frac{d^2y}{dx^2} = 13 > 0 \Rightarrow$ minimum

6b. $y = \int 3x^2 - 4x^{-2} - 11 \; dx$

$y = x^3 + \frac{4}{x} - 11x + c$

when $x = 2$, $y = 1$

$1 = 2^3 + \frac{4}{2} - 11(2) + c$

$1 = 8 + 2 - 22 + c \Rightarrow c = 13$

$y = x^3 + \frac{4}{x} - 11x + 13$

7a. $(\tan\theta + 1)(\sin^2\theta - 3\cos^2\theta) = 0$

$\tan\theta + 1 = 0$ or $\sin^2\theta - 3\cos^2\theta = 0$ $(\div \cos^2\theta)$

$\tan\theta = -1$

$\dfrac{\sin^2\theta}{\cos^2\theta} = 3$

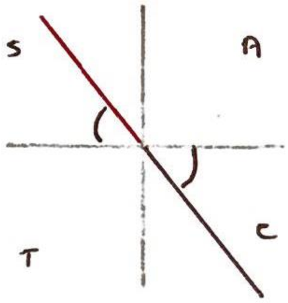
$\tan^2\theta = 3$

$\tan\theta = \pm\sqrt{3}$

7b. $0 \leq \theta \leq 180°$

$\tan\theta = -1$

P.V. $\theta = -45°$



$\theta = 135°$

$\tan\theta = +\sqrt{3}$

P.V. $\theta = 60°$

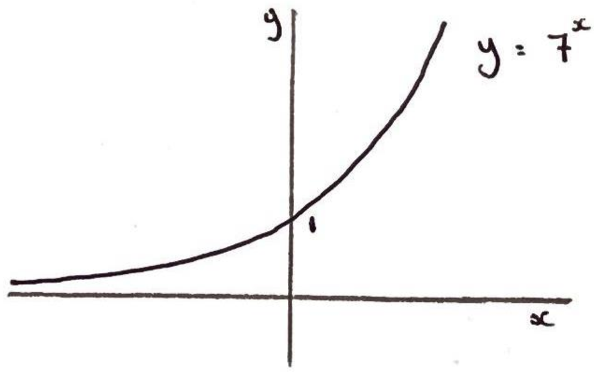
$\tan\theta = -\sqrt{3}$

P.V. $\theta = -60°$

$\theta = 120°$

$\theta = 60°, 120°, 135°$

8a.



8b. $y = 7^x$, $y = 7^{2x} - 12$

$7^{2x} - 12 = 7^x$

$7^{2x} - 7^x - 12 = 0$ let $y = 7^x$

$y^2 - y - 12 = 0$ $y^2 = 7^{2x}$

$(y-4)(y+3) = 0$

$7^x = 4, \Rightarrow y = 4$ or $7^x = -3$ ✗

8bi $x \log 7 = \log 4$

$x = \dfrac{\log 4}{\log 7}$ Since $7^x > 0 \ \forall \ x \in \mathbb{R}$

(See sketch in 8a)

$= 0.712$ (3sf)

9a. $\int_0^1 \log(x^2+1) \, dx$

$h = \dfrac{1-0}{4} = 1/4$

x	y
0	$\log 1$
0.25	$\log 17/16$
0.5	$\log 5/4$
0.75	$\log 25/16$
1	$\log 2$

$$\int \approx \frac{1}{2}\left(\frac{1}{4}\right)\left\{\left(\log 1 + \log 2 + 2\left(\log \frac{17}{16} + \log \frac{5}{4} + \log \frac{25}{16}\right)\right\}\right.$$

$= 0.117$ (3sf)

9b. $f(x) = 2\log x$

$2\log x \rightarrow 1 + 2\log x$

$f(x) \rightarrow 1 + f(x)$ Translation $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

9ci.

$$\log_{10}(10x^2) = \log_{10} 10 + \log_{10} x^2$$
$$= 1 + 2\log_{10} x$$

9cii.

$$2\log_{10} x \rightarrow \log_{10}(10x^2)$$
$$\log_{10} x^2 \rightarrow \log_{10}(\sqrt{10}x^2)$$

stretch s.f. $\frac{1}{\sqrt{10}}$ in x direction

9ciii.

$$\log(10x^2) = \log_{10}(x^2+1)$$
$$10x^2 = x^2+1$$
$$9x^2 = 1$$
$$x = \pm\sqrt{\frac{1}{9}}$$
$$= \pm\sqrt{\frac{1}{3}} \Rightarrow x = \frac{1}{3} \quad (\text{since } x > 0)$$

$$y = \log 10\left(10(1/3)^2\right)$$
$$= \log_{10}\left(\frac{10}{9}\right)$$

$$m = \frac{\log_{10}\left(\frac{10}{9}\right) - 0}{1/3 - 0}$$
$$= 3\log_{10}\left(\frac{10}{9}\right)$$
$$= \log_{10}\frac{10^3}{9^3}$$
$$= \log_{10}\frac{1000}{729}$$