AQA

A Level

A Level Maths

AQA Core Maths C1 June 2012 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

AQA June 12 C1

1.

$$\frac{5\sqrt{3}-6}{2\sqrt{3}+3} \times (2\sqrt{3}-3)$$

$$= \frac{(5\sqrt{3}-6)(2\sqrt{3}-3)}{(2\sqrt{3}+3)(2\sqrt{3}-3)} = \frac{10(3) - 15\sqrt{3} - 12\sqrt{3} + 18}{4(3) - 9}$$

$$= \frac{48 - 27\sqrt{3}}{3}$$

$$= 16 - 9\sqrt{3}$$

2ai.

$4x - 3y = 7$

$3y = 4x - 7$

$y = \frac{4}{3}x - \frac{7}{3} \Rightarrow m = \frac{4}{3}$

2aii.

$// \Rightarrow m = \frac{4}{3}$

$y - -5 = \frac{4}{3}(x-3) \quad \times 3$

$3y + 15 = 4x - 12$

$4x - 3y + \cancel{3}\ 27 = 0$

$\therefore \quad 4x - 3y = 27$

2b.

$4x - 3y = 7 \quad \times 2$

$3x - 2y = 4 \quad \times 3$

$8x - 6y = 14$ ①

$9x - 6y = 12$ ②

'② - ①' $\quad x = -2$

'sub $x = -2$ into $4x - 3y = 7$'

$-8 - 3y = 7$

$-3y = 15$

$y = -5$

$D(-2, -5)$

2c.

$$x = k-2 \quad , \quad y = 2k-3$$

$$4(k-2) - 3(2k-3) = 7$$

$$4k - 8 - 6k + 9 = 7$$

$$-2k = 6$$

$$k = -3$$

3ai.

$$p(x) = x^3 + 2x^2 - 5x - 6$$

$$p(-1) = (-1)^3 + 2(-1)^2 - 5(-1) - 6$$

$$= -1 + 2 + 5 - 6$$

$$= 0$$

$\therefore$ $(x+1)$ is a factor

3aii.

$$\begin{array}{r|l} & x^2 + x - 6 \\ \hline x+1 & x^3 + 2x^2 - 5x - 6 \\ & \underline{x^3 + x^2} \\ & x^2 - 5x \\ & \underline{x^2 + x} \\ & -6x - 6 \\ & \underline{-6x - 6} \\ & 0 \end{array}$$

$$p(x) = (x+1)(x^2 + x - 6)$$

$$= (x+1)(x+3)(x-2)$$

3b.

$$p(0) = 0^3 + 2(0)^2 - 5(0) - 6$$

$$= -6$$

$$p(1) = (1)^3 + 2(1)^2 - 5(1) - 6$$

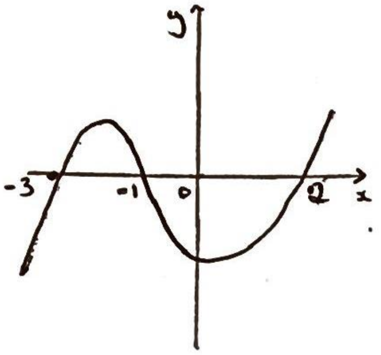
$$= 1 + 2 - 5 - 6$$

$$= -8$$

$$-6 > -8 \implies p(0) > p(1)$$

3c. $y = (x+1)(x+3)(x-2)$ roots at $-1, -3, 2$

$+x^3 \therefore$ N shape



4ai. $2(3xy + yx + 3x^2) = 32$

$4xy + 3x^2 = 16$

4aii. $V = 3x^2y$ $\qquad y = \dfrac{16 - 3x^2}{4x}$

$$V = 3x^2\left(\frac{16 - 3x^2}{4x}\right)$$

$$= \frac{48x^2 - 9x^3}{4}$$

$$= 12x^2 - \frac{9x^3}{4}$$

4b $\dfrac{dV}{dx} = 12 - \dfrac{27}{4}x^2$

4ci. stat when $\dfrac{dV}{dx} = 0$, $\quad x = \dfrac{4}{3}$, $\dfrac{dV}{dx} = 12 - \dfrac{27}{4}\left(\dfrac{4}{3}\right)^2$

$$= 12 - \frac{27}{4}\cdot\frac{16}{9}$$

$$= 12 - 12$$

$$= 0$$

$\therefore$ stat. point when $x = 4/3$

4cii.

$$\frac{d^2V}{dx^2} = -\frac{27}{2}x$$

when $x = \frac{4}{3}$, $\frac{d^2V}{dx^2} = -\frac{27}{2}\left(\frac{4}{3}\right)$

$$= -18$$

$$\frac{d^2V}{dx^2} < 0 \Rightarrow \text{maximum}$$

5ai.

$$x^2 - 3x + 5 = (x - 3/2)^2 - 9/4 + 5$$

$$= (x - 3/2)^2 + 11/4$$

5aii. minimum at $(+3/2, 11/4)$ $\Rightarrow$ $x = 3/2$ line of symmetry

5bi.

$y = x^2 - 3x + 5$, $y = x + 5$

$$x^2 - 3x + 5 = x + 5$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$x = 0$ or $x = 4$

at A $x = 0$,

$\therefore$ at B $x = 4$, $y = 4 + 5$
$= 9$

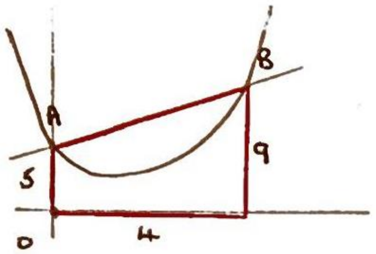
so B $(4, 9)$

5bii.

$$\int x^2 - 3x + 5 \; dx$$

$$= \frac{1}{3}x^3 - \frac{3}{2}x^2 + 5x + c$$

5biii.



A of trap $= \frac{1}{2}(5+9) \times 4$

$= 28$

A under curve $= \left[\frac{1}{3}x^3 - \frac{3}{2}x^2 + 5x\right]_0^4$

$= \frac{1}{3}(4)^3 - \frac{3}{2}(4)^2 + 5(4) - 0$

$= \frac{64}{3} - 24 + 20$

$= \frac{64}{3} - 4$

$= \frac{52}{3}$

$\therefore \quad R = 28 - \frac{52}{3}$

$= \frac{32}{3}$

6a. $(x-5)^2 + (y-8)^2 = 25$ (radius = x coord. of c)

6bi. $x=2, \quad y=12$

$(\cancel{x}\ 2-5)^2 + (12-8)^2$

$= 9 + 16$

$= 25 \Rightarrow (2,12)$ lies on circle

6bi. grad of $AC = \frac{12-8}{2-5} = -\frac{4}{3}$

$\therefore$ m of tangent $= \frac{3}{4}$ (since $\perp$)

$$y - 12 = \frac{3}{4}(x - 2) \quad \times 4$$

$$4y - 48 = 3x - 6$$

$$3x - 4y + 42 = 0$$

6ci. $m = (7, 12) \qquad c\,(5, 8)$

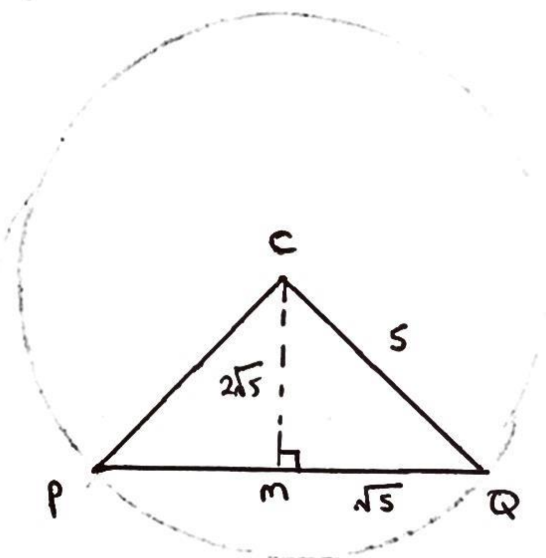
$$|cm| = \sqrt{(7-5)^2 + (12-8)^2}$$

$$= \sqrt{4 + 16}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$

6cii.



$$cm^2 = 5^2 - (\sqrt{5})^2$$

$$cm^2 = 20$$

$$cm = \sqrt{20}$$

$$= 2\sqrt{5}$$

$$A = \frac{1}{2} b \times h \qquad b = 2\sqrt{5} \quad (PQ)$$

$$h = 2\sqrt{5} \quad (cm)$$

$$A = \frac{1}{2}(2\sqrt{5} \times 2\sqrt{5})$$

$$= 10$$

7ai. y increasing when $\frac{dy}{dx} > 0$

$20x - 6x^2 - 16 > 0 \quad \Rightarrow \quad 6x^2 - 20x + 16 < 0$

$3x^2 - 10x + 8 < 0$

7aii. $(3x - 4)(x - 2) < 0$

c.v.s $x = 2$, $x = 4/3$

so $\frac{4}{3} < x < 2$

7bi. when $x = 2$

$\frac{dy}{dx} = 20(2) - 6(2)^2 - 16$

$= 40 - 24 - 16$

$= 0$

$\therefore$ parallel to x axis

7bii. when $x = 3$

$\frac{dy}{dx} = 20(3) - 6(3)^2 - 16$

$= 60 - 54 - 16$

$= -10$

$\therefore$ grad of norm $= \frac{1}{10}$

$y - -1 = \frac{1}{10}(x - 3)$

$10y + 10 = x - 3$

$10y = x - 13$

eq^n of tangent at P $\quad y - 3 = 0(x - 2) \quad$ (grad $= 0$)

$$y = 3$$

sub $y = 3$ into $10y = x - 13$

$$30 = x - 13$$

$$x = 43$$

$\therefore \quad R \ (43, 3)$