AQA

A Level

A Level Maths

AQA Core Maths C3 June 2011

Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

AQA June 11 C3

1a. $y = \ln(6x)$

when $y = 0$, $0 = \ln(6x)$

$6x = 1$ (Since $\ln 1 = 0$)

$x = 1/6$

1b. $\frac{dy}{dx} = \frac{6}{6x} = \frac{1}{x}$ $\left(\frac{d}{dx} \ln(f(x)) = \frac{f'(x)}{f(x)} \right)$

1c. $\int_1^7 \ln(6x)\,dx$ $h = \frac{7-1}{6} = 1$

x	y
1	ln6
2	ln12
3	ln18
4	ln24
5	ln30
6	ln36
7	ln42

$$\int \approx \frac{1}{3}(1)\left\{(\ln 6 + \ln 42) + 4(\ln 12 + \ln 24 + \ln 36) + 2(\ln 18 + \ln 30)\right\}$$

$= 18.4$ (3 s.f.)

2ai. $y = xe^{2x}$

$\frac{dy}{dx} = e^{2x} + 2xe^{2x}$

2aii. when $x = 1$, $\frac{dy}{dx} = e^2 + 2e^2$ $(1, e^2)$

$= 3e^2$

$y - e^2 = 3e^2(x - 1)$

2b.

$$y = \frac{2\sin 3x}{1+\cos 3x}$$

Quotient: $f = 2\sin 3x$ $\quad g = 1+\cos 3x$

$f' = 6\cos 3x$ $\quad g' = -3\sin 3x$

$$\frac{dy}{dx} = \frac{6\cos 3x(1+\cos 3x) - 2\sin 3x(-3\sin 3x)}{(1+\cos 3x)^2}$$

$$= \frac{6\cos 3x + 6\cos^2 3x + 6\sin^2 3x}{(1+\cos 3x)^2}$$

$$= \frac{6\cos 3x + 6(\cos^2 3x + \sin^2 3x)}{(1+\cos 3x)^2}$$

$$= \frac{6(\cos 3x + 1)}{(1+\cos 3x)^2} \qquad (\cos^2 3x + \sin^2 3x \equiv 1)$$

$$= \frac{6}{1+\cos 3x}$$

3a.

$y = \cos^{-1}(2x-1)$, $\quad y = e^x$

intersect when $\cos^{-1}(2x-1) = e^x$

let $f(x) = \cos^{-1}(2x-1) - e^x$

$f(0.4) = 0.2803\ldots$

$f(0.5) = -0.0779\ldots$

change of sign $\Rightarrow 0.4 < \alpha < 0.5$

3b.

$$\cos^{-1}(2x-1) = e^x$$

$$2x-1 = \cos(e^x)$$

$$2x = 1 + \cos(e^x)$$

$$\alpha = \frac{1}{2} + \frac{1}{2}\cos(e^x)$$

3c.

$$x_{n+1} = \frac{1}{2} + \frac{1}{2}\cos(e^{x_n})$$

$x_1 = 0.4$

$x_2 = 0.539$

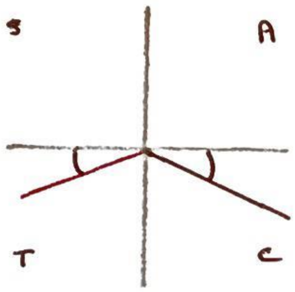
$x_3 = 0.428 \quad (3\text{ d.p.})$

4ai.

$\operatorname{cosec}\theta = -4 \qquad 0^\circ < \theta < 360^\circ$

$\sin\theta = -1/4$

P.V. $\theta = -14.478^\circ$



$\theta = 345.5^\circ, \; 194.5^\circ$

4aii.

$2\cot^2(2x+30^\circ) = 2 - 7\operatorname{cosec}(2x+30^\circ) \qquad 0 < x < 180^\circ$

$\cot^2\theta \equiv \operatorname{cosec}^2\theta - 1$

$2(\operatorname{cosec}^2(2x+30) - 1) = 2 - 7\operatorname{cosec}(2x+30)$

$2\operatorname{cosec}^2(2x+30) + 7\operatorname{cosec}(2x+30) - 4 = 0$

$(2\operatorname{cosec}^2(2x+30) - 1)(\operatorname{cosec}(2x+30) + 4) = 0$

$\operatorname{cosec}(2x+30) = \frac{1}{2}$ or $\operatorname{cosec}(2x+30^\circ) = -4$

$\sin(2x+30) = 2 \; \times \qquad \sin(2x+30^\circ) = -1/4$

$(\text{Since } -1 \leq \sin\theta \leq 1 \;\; \forall\, \theta \in \mathbb{R})$

$\Rightarrow \quad 2x + 30^\circ = 194.5^\circ \Rightarrow x = 82.3^\circ$

$\qquad\qquad = 345.5^\circ \Rightarrow x = 157.8^\circ$

4b. $y = \operatorname{cosec} x \rightarrow y = \operatorname{cosec}(2x + 30)$

$y = \operatorname{cosec} x \rightarrow y = \operatorname{cosec}(2x)$ stretch s.f. $\frac{1}{2}$ in x direction

$y = \operatorname{cosec}(2x) \rightarrow y = \operatorname{cosec}(2(x+15))$ translation $\begin{pmatrix} -15 \\ 0 \end{pmatrix}$

5a. $f(x) = x^2, \quad \forall x \in \mathbb{R}$

$g(x) = \dfrac{1}{2x+1}, \quad x \neq -1/2$

$f(x)$ is not 1-1 $\Rightarrow$ no inverse

5b. let $y = \dfrac{1}{2x+1}$

$2yx + y = 1$

$2yx = 1 - y$

$x = \dfrac{1-y}{2y}$

$\therefore \quad g^{-1}(x) = \dfrac{1-x}{2x}$

5c. range of $g^{-1}(x)$ = domain of $g(x)$

$\therefore \quad g^{-1}(x) \neq -1/2$

5d. $fg(x) = g(x)$

$\left(\dfrac{1}{2x+1}\right)^2 = \dfrac{1}{2x+1}$

$\Rightarrow \quad (2x+1)^2 = 2x+1$

$4x^2 + 4x + 1 = 2x + 1$

$4x^2 + 2x = 0$

$2x(2x+1) = 0$

$x = 0$ or $-1/2$ since $x \neq -1/2$

$x = 0$

6a.

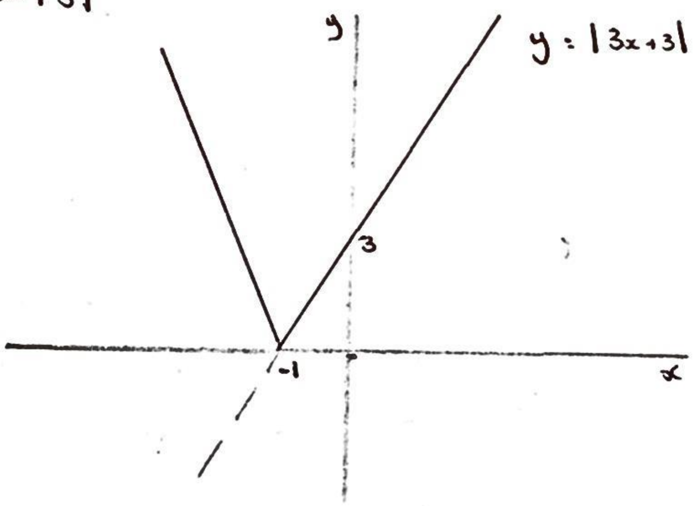
$$3\ln x = 4$$
$$\ln x = 4/3$$
$$x = e^{4/3}$$

6b.

$$3\ln x + \frac{20}{\ln x} = 19 \qquad (\times \ln x)$$
$$3\ln^2 x + 20 = 19\ln x$$
$$3\ln^2 x - 19\ln x + 20 = 0$$
$$(3\ln x - 4)(\ln x - 5) = 0$$
$$3\ln x = 4$$
$$\Rightarrow x = e^{4/3} \quad \text{or} \quad \ln x = 5$$
$$\Rightarrow x = e^5.$$

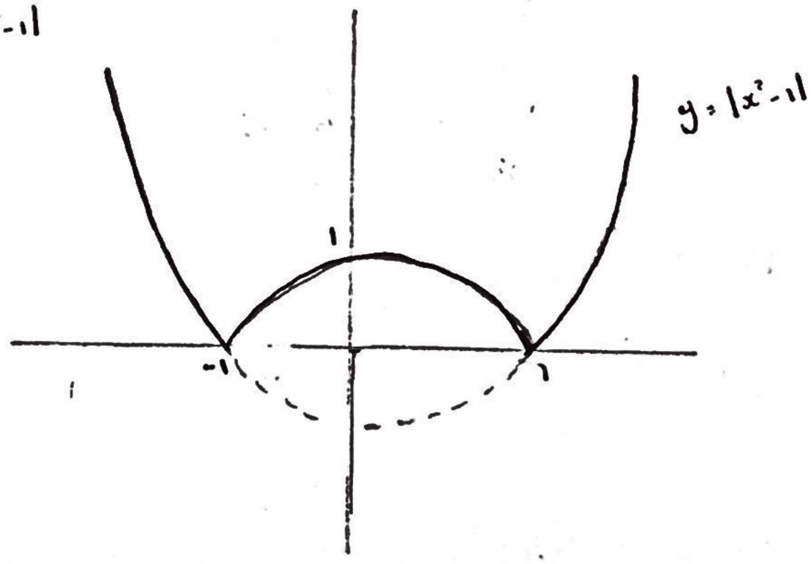
7ai.

$y = |3x+3|$



7aii.

$y = |x^2-1|$



7bi $|3x+3| = |x^2-1|$

$3x+3 = x^2-1$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

$x = 4$ or -1

$3x+3 = -(x^2-1)$

$3x+3 = 1-x^2$

$x^2+3x+2=0$

$(x+2)(x+1)=0$

$x = -2$ or -1

$\therefore \quad x = -2, -1, 4$

7bii. $|3x+3| < |x^2-1|$

4 intervals to check :

a $\quad x < -2 \quad$ is $\quad |3(-3)+3| < |(-3)^2-1|$

$6 < 8$ ✓

b $\quad -2 < x < -1, \quad$ is $\quad |3(-1.5)+3| < |(-1.5)^2-1|$

$3/2 < 5/4$ ×

c $\quad -1 < x < 4 \quad$ is $\quad |3(1)+3| < |1^2-1|$

$6 < 0$ ×

d $\quad x > 4 \quad$ is $\quad |3(5)+3| < |5^2-1|$

$18 < 24$ ✓

$\therefore$ if $|3x+3| < |x^2-1|$

$x < -2$ or $x > 4$.

8.

$$\int \frac{1}{(1+2\tan x)^2 \cos^2 x} \, dx$$

$u = 1 + 2\tan x$

$du = 2\sec^2 x \; dx$

$\therefore \; dx = \dfrac{du}{2\sec^2 x} = \dfrac{1}{2}\cos^2 x \; du$

$$\int \frac{1}{u^2 \cos^2 x} \cdot \frac{1}{2}\cos^2 x \; du$$

$$= \frac{1}{2} \int u^{-2} \, du$$

$$= \frac{1}{2} \cdot -u^{-1} + c$$

$$= -\frac{1}{2u} + c$$

$$= -\frac{1}{2+4\tan x} + c$$

9a.

$$\int x \ln x \; dx$$

Parts

$u = \ln x \qquad v' = x$

$u' = \dfrac{1}{x} \qquad v = \dfrac{1}{2}x^2$

$$= \frac{1}{2}x^2 \ln x - \int \frac{1}{x} \cdot \frac{1}{2}x^2 \; dx$$

$$= \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x \; dx$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$$

9b.

$$y = (\ln x)^2$$

$$\frac{dy}{dx} = 2 \cdot \frac{1}{x} (\ln x)^1$$

$$= \frac{2}{x} \ln x$$

9c. $V = \pi \int_1^e y^2 \, dx$ $\qquad y = \sqrt{x} \ln x$

$y^2 = x \ln^2 x$

$V = \pi \int_1^e x (\ln x)^2 \, dx$

Parts: $u = (\ln x)^2$ $\quad v' = x$

$u' = \frac{2}{x} \ln x$ $\quad v = \frac{1}{2}x^2$

$= \pi \left[\frac{1}{2}x^2 (\ln x)^2 - \int \frac{2}{x} \ln x \cdot \frac{1}{2}x^2 \, dx \right]$

$= \pi \left[\frac{1}{2}x^2 (\ln x)^2 - \int x \ln x \, dx \right]$

$= \pi \left[\frac{1}{2}x^2 (\ln x)^2 - \left(\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 \right) \right]_1^e$

$= \pi \left[\frac{1}{2}x^2 (\ln x)^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2 \right]_1^e$

$= \frac{\pi}{4} \left[\left(2e^2 (\ln e)^2 - 2e^2 \ln e + e^2 \right) - \left(2(1)^2 (\ln 1)^2 - 2(1)^2 \ln 1 + 1 \right) \right]$

$= \frac{\pi}{4} \left[\left(2e^2 - 2e^2 + e^2 \right) - \left(0 - 0 + 1 \right) \right]$

$= \frac{\pi}{4} \left(e^2 - 1 \right)$