AQA

A Level

A Level Maths

AQA Core Maths C2 June 2011 Model Solutions

Name:

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Total Marks:

AQA June 11 C2

1a. Sine Rule : $\frac{\sin\theta}{10} = \frac{\sin 54}{9}$

$\theta = \sin^{-1}\left(\frac{10\sin 54}{9}\right)$

$= 64.01487...$

$= 64°$ (to nearest degree)

1b. Area $= \frac{1}{2}ab\sin C$

$C = 180 - 54 - 64.014...$

$= 61.985...$

$= \frac{1}{2}(10)(9)\sin 61.985$

$= 39.727...$

$= 40$ to nearest cm^2

2a. $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}(6)^2(0.5)$

$= 9$

2bi. $l = r\theta$

$= 6(0.5)$

$= 3$

2bii. $P = 2r + l$

$= 15$

$= 5 \times l \Rightarrow k = 5$

3a. $(2+x^2)^3 = 2^3 + {}^3C_1 2^2(x^2) + {}^3C_2 2(x^2)^2 + (x^2)^3$

$= 8 + 12x^2 + 6x^4 + x^6$

$\therefore q = 6$

2bi. $\int \frac{(2+x^2)^3}{x^4}\,dx = \int \frac{8 + 12x^2 + 6x^4 + x^6}{x^4}\,dx$

$= \int 8x^{-4} + 12x^{-2} + 6 + x^2\,dx$

$= -\frac{8}{3}x^{-3} - 12x^{-1} + 6x + \frac{1}{3}x^3 + c$

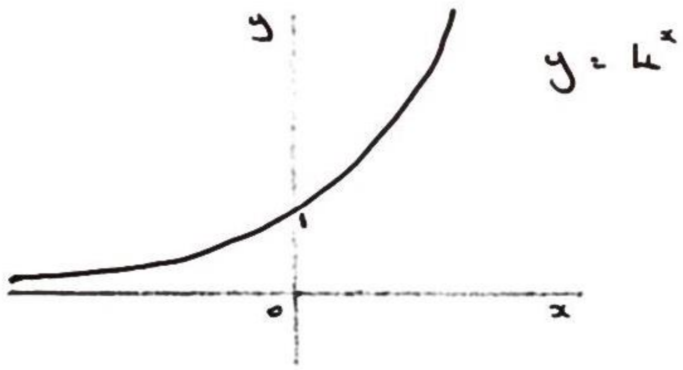
2bii. $\int_1^2 \frac{(2+x^2)^3}{x^4}\,dx = \left[-\frac{8}{3}x^{-3} - 12x^{-1} + 6x + \frac{1}{3}x^3\right]_1^2$

$= \left(-\frac{8}{3}(2)^{-3} - 12(2)^{-1} + 6(2) + \frac{1}{3}(2)^3\right) - \left(-\frac{8}{3}(1)^{-3} - 12(1)^{-1} + 6(1) + \frac{1}{3}(1)^3\right)$

$= \frac{25}{3} - \left(-\frac{25}{3}\right)$

$= \frac{50}{3}$

4a



4b. $y = 4^x \rightarrow y = 4^x - 5$

$f(x) \rightarrow f(x) - 5$ translation 5 units right

4ci. $4^x - 2^{x+2} - 5 = 0$ $\quad Y = 2^x$

$Y^2 - 2^2Y - 5 = 0$ $\quad 4^x = (2^2)^x = 2^{2x} = (2^x)^2 = Y^2$

$Y^2 - 4Y - 5 = 0$

4cii. $(y-5)(y+1) = 0$

$y = 5$

$2^x = 5$ or $2^x + 1 = 0$

$\log 2^x = \log 5$ $\quad 2^x = -1$ ×

$x \log 2 = \log 5$ $\quad (2^x > 0 \;\; \forall x \in \mathbb{R})$

$x = \dfrac{\log 5}{\log 2}$

$x = 2.322$ (3dp.)

5a. $y = 6x - 2x^{3/2}$

$\dfrac{dy}{dx} = 6 - 3x^{1/2}$

5bi. At M, $\dfrac{dy}{dx} = 0$

$3x^{1/2} = 6$

$x^{1/2} = 2$

$x = 4$

when $x = 4$, $y = 6(4) - 2(4)^{3/2}$

$= 8$

$\therefore$ M at $(4, 8)$

5bii. grad at M $= 0$ $\Rightarrow$ grad of normal $= -\dfrac{1}{0} = \infty$

$\therefore x = 4$

5ci. At P, $x = 9/4$, $\dfrac{dy}{dx} = 6 - 3\left(\frac{9}{4}\right)^{1/2}$

$= 3/2$

$\therefore$ m of normal $= -\dfrac{2}{3}$ (since $\perp$)

$$y - \frac{27}{4} = -\frac{2}{3}(x - 9/4) \qquad \times 3$$

$$3y - \frac{81}{4} = -2x + \frac{9}{2} \qquad \times 4$$

$$12y - 81 = -8x + 18$$

$$8x + 12y = 99$$

5cii.

$x = 4$ ① $\qquad 8x + 12y = 99$ ②

'Sub ① into ②'

$$8(4) + 12y = 99$$
$$12y = 67$$
$$y = \frac{67}{12}$$

R at $(4, 67/12)$

6a.

$$\int_0^2 \sin x \, dx \qquad h = \frac{2-0}{4} = \frac{1}{2}$$

x	y
0	0
0.5	$\sin(0.5)$
1	$\sin(1)$
1.5	$\sin(1.5)$
2	$\sin(2)$

$$\int \approx \frac{1}{2}\left(\frac{1}{2}\right)\left\{(0 + \sin 2) + 2(\sin 0.5 + \sin 1 + \sin 1.5)\right\}$$

$$= 1.39 \quad (3 sf)$$

6b.

$$y = \sin x \rightarrow y = 2\sin x$$

$f(x) \rightarrow 2f(x)$ Stretch s.f. 2 in y direction

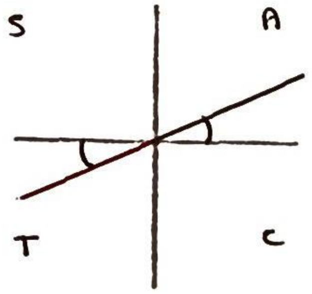
6c.

$$2\sin x = \cos x$$

$$\frac{2\sin x}{\cos x} = 1$$

$$\tan x = \frac{1}{2} \qquad 0 \leq x \leq 2\pi$$

P.V. $x = 0.4636...$



$x = 0.464^c,\ 3.61^c$ (3 s.f.)

7a. $U_{n+1} = pU_n + q$

$U_1 = 60, \quad U_2 = 48$

$U_2 = pU_1 + q$

$48 = 60p + q$ ①

$l = 12$

$\therefore\ l = pl + q$

$12 = 12p + q$ ②

'② − ①' $\therefore$ $\cancel{\frac{48}{12}}$ $36 = 48p \Rightarrow p = \frac{3}{4}$

Sub $p = 3/4$ into ②'

$12 = 12\left(\frac{3}{4}\right) + q$

$q = 3$

7b. $U_3 = \frac{3}{4}U_2 + 3$

$= \frac{3}{4}(48) + 3$

$= 39$

8. $(3\sin x + \cos x)^2 + (\cancel{3}\sin x - 3\cos x)^2$

$= (9\sin^2 x + 6\sin x\cos x + \cos^2 x) + (\sin^2 x - 6\sin x\cos x + 9\cos^2 x)$

$= 10\sin^2 x + 10\cos^2 x$

$= 10(\sin^2 x + \cos^2 x)$ $\quad (\sin^2 x + \cos^2 x = 1)$

$= 10$

9a. GP. $a = 12$, $r = 3/8$

$S_\infty = \frac{a}{1-r} = \frac{12}{1-3/8} = \frac{96}{5}$

9b. $u_6 = ar^5 = 12 \times \left(\frac{3}{8}\right)^5$

$= \frac{12 \times 3^5}{8^5}$ $\quad 12 = 2^2 \times 3$

$= \frac{2^2 \times 3 \times 3^5}{2^{15}}$ $\quad 8^5 = (2^3)^5 = 2^{15}$

$= \frac{2^2 \times 3^6}{2^{15}}$

$= \frac{3^6}{2^{13}}$

9c. $u_n = ar^{n-1}$

$= 12\left(\frac{3}{8}\right)^{n-1}$

$\log_a u_n = \log_a\left(12\left(\frac{3}{8}\right)^{n-1}\right)$

$= \log_a 12 + (n-1)\log_a\left(\frac{3}{8}\right)$

$= 2\log 2 + \log 3 + (n-1)\log_a\left(\frac{3}{8}\right)$ $\quad \log 12 = \log 4 + \log 3 = 2\log 2 + \log 3$

$= 2\log 2 + \log 3 + n\log_a\left(\frac{3}{8}\right) - \log_a\left(\frac{3}{8}\right)$

$= 2\log 2 + \cancel{\log 3} + n\log_a 3 - n\log_a 8 - \cancel{\log_a 3} + \log_a 8$

$= 2\log 2 + n\log_a 3 - n\log_a 2^3 + \log_a 2^3$

$= 2\log 2 + n\log_a 3 - 3n\log_a 2 + 3\log_a 2$

$= n\log_a 3 + \log 2(2 - 3n + 3)$

$= n\log_a 3 + (5-3n)\log 2$

$= n\log_a 3 - (3n-5)\log 2$