AQA

A Level

A Level Maths

AQA Core Maths C1 June 2011 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

1a.

$7x + 3y = 13$

$3y = 13 - 7x$

$y = \frac{13}{3} - \frac{7}{3}x \quad \Rightarrow \quad m = -7/3$

1bi.

$/\!/ \quad \Rightarrow \quad m = -7/3 \qquad C\ (-1, 3)$

$y - 3 = -\frac{7}{3}(x - -1)$

$3y - 9 = -7x - 7$

$3y + 7x = 2$

1bii.

M.d $= (1\frac{1}{2}, -1) \qquad C\ (-1, 3)$

$-1 \rightarrow 1\frac{1}{2} = +2\frac{1}{2} \quad ; \quad 1\frac{1}{2} + 2\frac{1}{2} = 4$

$3 \rightarrow -1 = -4 \quad ; \quad -1 - 4 = -5$

$\therefore \quad A\ (4, -5)$

1c.

$7x + 3y = 13 \quad \times 3 \qquad 21x + 9y = 39 \quad ①$

$3x + 2y = 12 \quad \times 7 \qquad 21x + 14y = 84 \quad ②$

'② - ①' $\Rightarrow$ $5y = 45$

$y = 9$

'sub $y = 9$ into $3x + 2y = 12$

$3x + 18 = 12$

$3x = -6$

$x = -2$

$\therefore$ intersect at $(-2, 9)$

2a. $\sqrt{48} = \sqrt{16\times3} = 4\sqrt{3}$

2ai. $\dfrac{\sqrt{48}+2\sqrt{27}}{\sqrt{12}}$;

$\sqrt{48} = 4\sqrt{3}$

$2\sqrt{27} = 2\sqrt{9\times3} = 6\sqrt{3}$

$\sqrt{12} = \sqrt{4\times3} = 2\sqrt{3}$

$\dfrac{4\sqrt{3}+6\sqrt{3}}{2\sqrt{3}}$

$= \dfrac{10\sqrt{3}}{2\sqrt{3}}$

$= 5$

2b. $\dfrac{1-5\sqrt{5}}{3+\sqrt{5}} \times (3-\sqrt{5})$

$\dfrac{(1-5\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})} = \dfrac{3-\sqrt{5}-15\sqrt{5}+5(5)}{9-5}$

$= \dfrac{28-16\sqrt{5}}{4}$

$= 7-4\sqrt{5}$

3a. $V = \dfrac{t^3}{4} - 3t + 5$

$\dfrac{dV}{dt} = \dfrac{3}{4}t^2 - 3$

3bi. when $t=1$, $\dfrac{dV}{dt} = \dfrac{3}{4}(1)^2 - 3 = \dfrac{3}{4} - 3 = -9/4$

3bi. $\dfrac{dV}{dt} = -9/4 < 0 \Rightarrow$ decreasing

3ci. stat when $\frac{dV}{dt} = 0$

$$\frac{3}{4}t^2 - 3 = 0$$
$$3t^2 = 12$$
$$t^2 = 4$$
$$t = \pm 2 \quad \Rightarrow \quad t = 2$$

3cii

$$\frac{d^2V}{dt^2} = \frac{3}{2}t$$

when $t = 2$, $\frac{d^2V}{dt^2} = \frac{3}{2}(2) = 3$

$$\frac{d^2V}{dt^2} > 0 \Rightarrow \text{minimum}$$

4a.

$$x^2 + 5x + 7 = (x + 5/2)^2 - \frac{25}{4} + 7$$
$$= (x + 5/2)^2 - \frac{25}{4} + \frac{28}{4}$$
$$= (x + 5/2)^2 + 3/4$$

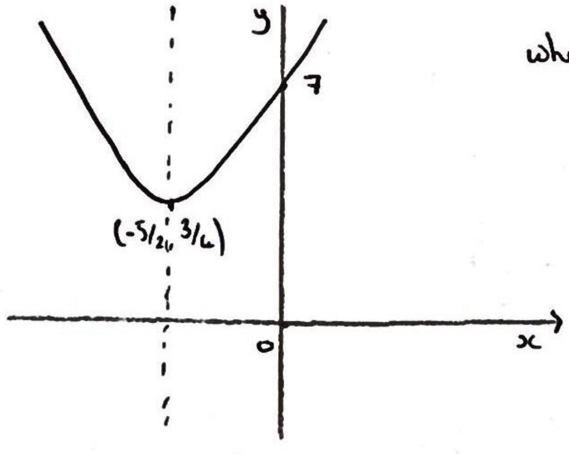
4bi

$$y = (x + 5/2)^2 + 3/4$$

$\therefore$ vertex at $(-5/2, 3/4)$

4bii. $x = -5/2$

4biii.



when $x = 0$

$y = 0^2 + 5(0) + 7$

so crosses at $(0, 7)$

4c. $y = x^2 \rightarrow y = (x + 5/2)^2 + 3/4$

$\therefore$ translation $5/2$ left and up $3/4$

5a.

$p(x) = x^3 - 2x^2 + 3$

$p(3) = (3)^3 - 2(3)^2 + 3$

$= 27 - 18 + 3$

$= 12 \quad \therefore$ remainder $= 12$

5b

$p(-1) = (-1)^3 - 2(-1)^2 + 3$

$= -1 - 2 + 3$

$= 0 \quad \therefore (x+1)$ is a factor

5ci.

$$
\begin{array}{r|l}
 & x^2 - 3x + 3 \\
x + 1 & x^3 - 2x^2 + 0x + 3 \\
 & \underline{x^3 + x^2} \\
 & -3x^2 + 0x \\
 & \underline{-3x^2 - 3x} \\
 & 3x + 3 \\
 & \underline{3x + 3} \\
 & 0
\end{array}
$$

$p(x) = (x+1)(x^2 - 3x + 3)$

5cii

$(x+1)(x^2 - 3x + 3) = 0$

$x + 1 = 0 \Rightarrow x = -1$

or $x^2 - 3x + 3 = 0$

disc. $(-3)^2 - 4(1)(3)$

$= 9 - 12$

$= -3$

$-3 < 0 \Rightarrow$ no real roots

6a.

$$\int_{-1}^{1} x^3 - 2x^2 + 3 \quad dx$$

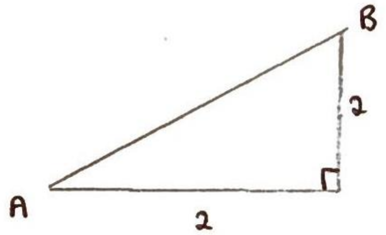
$$= \left[\tfrac{1}{4}x^4 - \tfrac{2}{3}x^3 + 3x \right]_{-1}^{1}$$

$$= \left(\tfrac{1}{4}(1)^4 - \tfrac{2}{3}(1)^3 + 3(1) \right) - \left(\tfrac{1}{4}(-1)^4 - \tfrac{2}{3}(-1)^3 + 3(-1) \right)$$

$$= \tfrac{1}{4} - \tfrac{2}{3} + 3 - \tfrac{1}{4} - \tfrac{2}{3} + 3$$

$$= 4\,{}^{2}/_{3}$$

6b.



$$A \text{ of } \Delta = \frac{2 \times 2}{2} = 2$$

Shaded Area = Area under curve − A of Δ

$$= 4\,{}^{2}/_{3} - 2$$

$$= 2\,{}^{2}/_{3}$$

7a.

$$2(4 - 3x) > 5 - 4(x+2)$$

$$8 - 6x > 5 - 4x - 8$$

$$11 > 2x$$

$$x < {}^{11}/_{2}$$

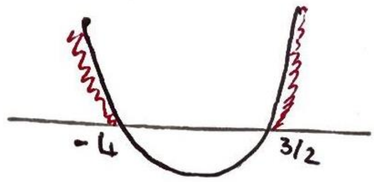
7b.

$$2x^2 + 5x \geq 12$$

$$2x^2 + 5x - 12 \geq 0$$

$$(2x - 3)(x + 4) \geq 0$$

c.v.s $\quad x = {}^{3}/_{2} \,, \quad x = -4$



$$x \leq -4$$

$$x \geq {}^{3}/_{2}$$

8a. C $(3,-8)$ radius $= 10$

$(x-3)^2 + (y+8)^2 = 100$

8b when curve crosses the x axis, $y = 0$

$(x-3)^2 + 8^2 = 100$

$(x-3)^2 = 36$

$x - 3 = \pm 6$

$x = 3 \pm 6$

$\therefore \quad x = 9$ or $x = -3$

8c. m of tangent $= 5/2 \Rightarrow$ m of CA $= -\frac{2}{5}$ (since $\perp$)

$y - -8 = -\frac{2}{5}(x-3)$

$\times 5$

$5y + 40 = -2x + 6$

$2x + 5y + 34 = 0$

8di. $y = 2x + 1$, sub into circle:

$(x-3)^2 + (2x+1+8)^2 = 100$

$(x-3)^2 + (2x+9)^2 = 100$

$x^2 - 6x + 9 + 4x^2 + 36x + 81 = 100$

$5x^2 + 30x - 10 = 0$

$x^2 + 6x - 2 = 0$

8dii

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-2)}}{2} = \frac{-6 \pm \sqrt{44}}{2}$$

$$= \frac{-6 \pm 2\sqrt{11}}{2} = -3 \pm \sqrt{11}$$