AQA

A Level

A Level Maths

AQA Core Maths C3 June 2010 Model Solutions

Name:

MME

Mathsmadeeasy.co.uk

Total Marks:

AQA June 10 C3

1a. $y = 3^x$, $y = 10 - x^3$

intersect when $3^x = 10 - x^3$

let $f(x) = 3^x + x^3 - 10$

$f(1) = -6$

$f(2) = 7$

change of sign $\Rightarrow 1 < \alpha < 2$

1bi. $3^x = 10 - x^3$

$x^3 = 10 - 3^x$

$x = \sqrt[3]{10 - 3^x}$

1bii. $x_{n+1} = \sqrt[3]{10 - 3^{x_n}}$

$x_1 = 1$

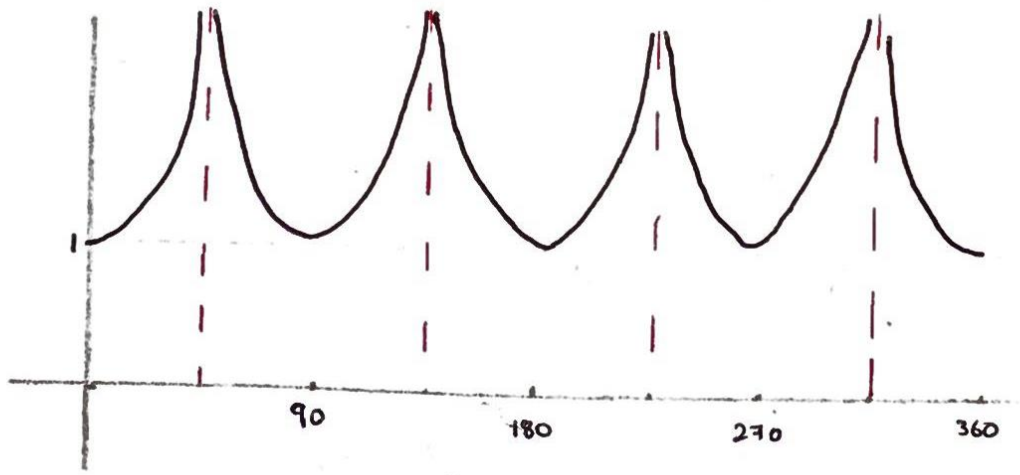
$x_2 = 1.913$

$x_3 = 1.221$

2ai. $y = \sec x$, when $x = 0$, $y = \sec 0 = \frac{1}{\cos(0)} = 1$ A $(0,1)$

2aii.

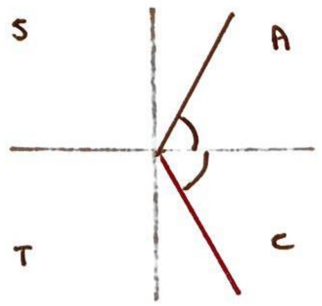
$y = |\sec 2x|$



2b. $\sec x = 2$ $0 \leq x \leq 360$

$\cos x = 1/2$

P.V. $x = 60°$



$x = 60°, 300°$

2c. $|\sec(2x-10)| = 2$ $0 \leq x \leq 180$

$\sec(2x-10) = 2$

$\cos(2x-10) = 1/2$

$2x-10 = 60° \Rightarrow x = 35°$

$300 \Rightarrow x = 155°$

or $\sec(2x-10) = -2$

$\cos(2x-10) = -1/2$

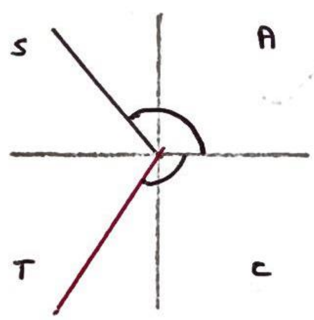
$0 \leq x \leq 180$

let $\phi = 2x-10$ $0 \leq 2x \leq 360$

$-10 \leq \phi \leq 350$

$\cos\phi = -1/2$

P.V. $\phi = 120°$



$\phi = 120°, 240°$

$2x-10 = 120° \Rightarrow x = 65°$

$2x-10 = 240° \Rightarrow x = 125°$

3ai.

$$y = \ln(5x-2)$$

$$\frac{dy}{dx} = \frac{5}{5x-2}$$

3aii.

$$y = \sin 2x$$

$$\frac{dy}{dx} = 2\cos 2x$$

3bi.

$$f(x) = \ln(5x-2) \qquad x \geq 1/2$$

$$g(x) = \sin 2x \qquad -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$$

$$f(1/2) = \ln(5(1/2)-2) = \ln(1/2)$$

$$\therefore \quad f(x) \geq \ln(1/2)$$

3bii.

$$gf(x) = g(\ln(5x-2))$$

$$= \sin(2\ln(5x-2))$$

3biii.

$$gf(x) = 0 \quad \Rightarrow \quad \sin(2\ln(5x-2)) = 0$$

$$2\ln(5x-2) = 0 \qquad (\div 2)$$

$$\ln(5x-2) = 0$$

$$5x-2 = 1$$

$$x = 3/5$$

3biv.

Let $y = \sin 2x$

$$2x = \sin^{-1}(y)$$

$$x = \frac{1}{2}\sin^{-1}(y)$$

$$\therefore \quad g^{-1}(x) = \frac{1}{2}\sin^{-1}(x)$$

4a. $\int_{0.5}^{2} \frac{x}{1+x^3} \, dx$ $\qquad h = \frac{2-0.5}{6} = 0.25$

x	y
0.5	$4/9$
0.75	$48/91$
1	$1/2$
1.25	$80/189$
1.5	$12/35$
1.75	$112/407$
2	$2/9$

$$\int \approx \frac{1}{3}(0.25)\left\{\left(\frac{4}{9}+\frac{2}{9}\right)+4\left(\frac{48}{91}+\frac{80}{189}+\frac{112}{407}\right)+2\left(\frac{1}{2}+\frac{12}{35}\right)\right\}$$

$$= 0.605 \quad (3 \text{ s.f.})$$

4b. $\int_{0}^{1} \frac{x^2}{1+x^3} \, dx$

$$= \frac{1}{3}\int_{0}^{1} \frac{3x^2}{1+x^3} \, dx$$

$$= \frac{1}{3}\left[\ln|1+x^3|\right]_0^1$$

$$= \frac{1}{3}(\ln 2 - \ln 1)$$

$$= \frac{1}{3}\ln 2$$

5a. $10\operatorname{cosec}^2 x = 16 - 11\cot x$

$10(\cot^2 x + 1) = 16 - 11\cot x$ $\qquad (\operatorname{cosec}^2 x \equiv \cot^2 x + 1)$

$10\cot^2 x + 10 = 16 - 11\cot x$

$10\cot^2 x + 11\cot x - 6 = 0$

5b. $(5\cot x - 2)(2\cot x + 3) = 0$

$\cot x = 2/5$ or $\cot x = -3/2$

$\therefore \tan x = 5/2$ or $-2/3$

6a $y = \frac{\ln x}{x}$

when $y = 0$, $0 = \frac{\ln x}{x}$ $(\times x)$

$0 = \ln x$

$x = 1$

6b. $y = \frac{\ln x}{x}$ Quotient : $f = \ln x$ $g = x$

$f' = 1/x$ $g' = 1$

$$\frac{dy}{dx} = \frac{\frac{1}{x} \cdot x - 1 \cdot \ln x}{x^2}$$

$$= \frac{1 - \ln x}{x^2}$$

at stat. pt. $\frac{dy}{dx} = 0$

$$\frac{1 - \ln x}{x^2} = 0$$

$1 - \ln x = 0$

$\ln x = 1$

$x = e$

when $x = e$, $y = \frac{\ln e}{e} = \frac{1}{e}$ B $(e, \frac{1}{e})$

6c. when $x = e^3$, $\frac{dy}{dx} = \frac{1 - \ln(e^3)}{(e^3)^2}$

$$= \frac{-2}{e^6}$$

$\therefore$ grad of normal $= \frac{e^6}{2}$ (since $\perp$)

7ai. $\int x\cos 4x \, dx$ Parts: $u = x$, $u' = 1$, $v' = \cos 4x$, $v = \frac{1}{4}\sin 4x$

$= \frac{x}{4}\sin 4x - \int \frac{1}{4}\sin 4x \, dx$

$= \frac{x}{4}\sin 4x + \frac{1}{16}\cos 4x + c$

7aii. $\int x^2 \sin 4x \, dx$ Parts: $u = x^2$, $u' = 2x$, $v' = \sin 4x$, $v = -\frac{1}{4}\cos 4x$

$= -\frac{x^2}{4}\cos 4x - \int 2x \cdot -\frac{1}{4}\cos 4x \, dx$

$= -\frac{x^2}{4}\cos 4x + \frac{1}{2}\int x\cos 4x \, dx$

$= -\frac{x^2}{4}\cos 4x + \frac{1}{2}\left(\frac{x}{4}\sin 4x + \frac{1}{16}\cos 4x\right) + c$ (from 7ai)

$= -\frac{x^2}{4}\cos 4x + \frac{1}{8}x\sin 4x + \frac{1}{32}\cos 4x + c$

7b $V = \pi\int_0^{0.2} y^2 \, dx$

$y = 8x\sqrt{\sin(4x)}$

$y^2 = 64x^2\sin 4x$

$V = 64\pi\int_0^{0.2} x^2\sin 4x \, dx$

$= 64\pi\left[-\frac{x^2}{4}\cos 4x + \frac{1}{8}x\sin 4x + \frac{1}{32}\cos 4x\right]_0^{0.2}$

$= 64\pi\left(\left(-\frac{0.2^2}{4}\cos(0.8) + \frac{1}{8}(0.2)\sin(0.8) + \frac{1}{32}\cos(0.8)\right) - \left(-0 + 0 + \frac{1}{32}\cos(0)\right)\right)$

$= 0.299$ (3 s.f.)

8a. $y = e^x \rightarrow y = e^{2x}$ stretch s.f. $\frac{1}{2}$ in x direction

$y = e^{2x} \rightarrow y = e^{2x} - 1$ translation $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

8b. $y = 4e^{-2x} + 2$

at A, $x = 0$, $y = 4e^{-0} + 2$

$= 6$ A $(0, 6)$

8ci. B when $4e^{-2x} + 2 = e^{2x} - 1$ $(\times e^{2x})$

$4 + 2e^{2x} = (e^{2x})^2 - e^{2x}$

$4 + 3e^{2x} = (e^{2x})^2$

$(e^{2x})^2 - 3e^{2x} - 4 = 0$

8cii. Let $x = e^{2x}$: $x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$\therefore e^{2x} = 4$ or $e^{2x} = -1$ $\times$

$2x = \ln 4$ (since $e^{2x} > 0 \ \forall x \in \mathbb{R}$)

$x = \frac{1}{2}\ln 4$

$= \ln 2$

8d. Shaded Area $= \int_0^{\ln 2} 4e^{-2x} + 2 \, dx - \int_0^{\ln 2} e^{2x} - 1 \, dx$

$= \int_0^{\ln 2} 4e^{-2x} - e^{2x} + 3 \, dx$

$= \left[-2e^{-2x} - \frac{1}{2}e^{2x} + 3x \right]_0^{\ln 2}$

$= \left(-2e^{-2\ln 2} - \frac{1}{2}e^{2\ln 2} + 3\ln 2 \right) - \left(-2e^0 - \frac{1}{2}e^0 + 0 \right)$

$= \left(-\frac{1}{2} - 2 + 3\ln 2 \right) + 2 + \frac{1}{2} = 3\ln 2$