AQA

A Level

A Level Maths

AQA Core Maths C2 June 2010
Model Solutions

Name:

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Total Marks:

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AQA June 10 C2

1a. $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}(8)^2(1.4)$

$= 44.8$

1bi. $P = 2r + l$

$l = r\theta$

$= 8(1.4)$

$= 11.2$

$P = 2(8) + 11.2$

$= 27.2$

1bii. $P = C$

$27.2 = 2\pi x$

$x = \frac{27.2}{2\pi}$

$= 4.33 \quad (3\,sf)$

2a. $u_{n+1} = 6 + \frac{2}{5}u_n \qquad u_1 = 2$

$u_2 = 6 + \frac{2}{5}u_1$

$= 6 + \frac{2}{5}(2)$

$= 6.8$

$u_3 = 6 + \frac{2}{5}u_2$

$= 6 + \frac{2}{5}(6.8)$

$= 8.72$

2b $l = 6 + \frac{2}{5}l$

$5l = 30 + 2l \Rightarrow 3l = 30, \quad l = 10$

3a Sine Rule : $\frac{\sin\theta}{6} = \frac{\sin 150}{15}$

$\theta = \sin^{-1}\left(\frac{6\sin 150}{15}\right)$

$= 11.5369...$

$= 11.5$ (to 1dp)

3b. $A\hat{B}C = 180 - 150 - 11.5...$

$= 18.463...$

Area $= \frac{1}{2}ab\sin C$

$= \frac{1}{2}(6)(15)\sin 18.463...$

$= 14.3$ (3 s.f.)

4a. $\left(1-\frac{1}{x^2}\right)^3 = 1^3 + {}^3C_1 1^2\left(-\frac{1}{x^2}\right) + {}^3C_2 1\left(-\frac{1}{x^2}\right)^2 + \left(-\frac{1}{x^2}\right)^3$

$= 1 - \frac{3}{x^2} + \frac{3}{x^4} - \frac{1}{x^6}$

$p = -3,\ q = 3$

4bi: $\int\left(1-\frac{1}{x^2}\right)^3 dx = \int 1 - 3x^{-2} + 3x^{-4} - x^{-6}\ dx$

$= x + 3x^{-1} - x^{-3} + \frac{1}{5}x^{-5} + c$

4bii. $\int_{1/2}^{1}\left(1-\frac{1}{x^2}\right)^3 dx = \left[x + 3x^{-1} - x^{-3} + \frac{1}{5}x^{-5}\right]_{1/2}^{1}$

$= \left(1 + 3(1)^{-1} - (1)^{-3} + \frac{1}{5}(1)^{-5}\right) - \left(\frac{1}{2} + 3\left(\frac{1}{2}\right)^{-1} - \left(\frac{1}{2}\right)^{-3} + \frac{1}{5}\left(\frac{1}{2}\right)^{-5}\right)$

$= \frac{16}{5} - \frac{49}{10}$

$= -\frac{17}{10}$

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5a. G.P. $a = 10$ $\quad S_\infty = 50$

$$S_\infty = \frac{a}{1-r}$$

$$50 = \frac{10}{1-r}$$

$$50 - 50r = 10$$

$$50r = 40$$

$$r = 4/5$$

5aii. $u_2 = ar = 10(4/5) = 8$

5bi. AP

$$u_4 = a + 3d = 10 \quad ①$$

$$u_8 = a + 7d = 8 \quad ②$$

'② − ①' $\quad 4d = -2$

$$d = -1/2$$

5bii.

$$a + 7(-\tfrac{1}{2}) = 8$$

$$a = 8 - 7(-1/2)$$

$$= \frac{23}{2}$$

$$\sum_{n=1}^{40} u_n = S_{40} = \frac{40}{2}\left(2a + 39d\right)$$

$$= 20\left(2\left(\tfrac{23}{2}\right) + 39\left(-\tfrac{1}{2}\right)\right)$$

$$= 70$$

6a.

$$y = \frac{x^3 + \sqrt{x}}{x}$$

$$= \frac{x^3}{x} + \frac{x^{1/2}}{x}$$

$$= x^2 + x^{-1/2}$$

6bi. $\frac{dy}{dx} = 2x - \frac{1}{2}x^{-3/2}$

6bii. when $x = 1$, $\frac{dy}{dx} = 2(1) - \frac{1}{2}(1)^{-3/2}$

$= \frac{3}{2}$

$\therefore$ m of normal $= -\frac{2}{3}$ (since $\perp$)

when $x = 1$, $y = \frac{1^3 + \sqrt{1}}{1} = 2$ $(1,2)$

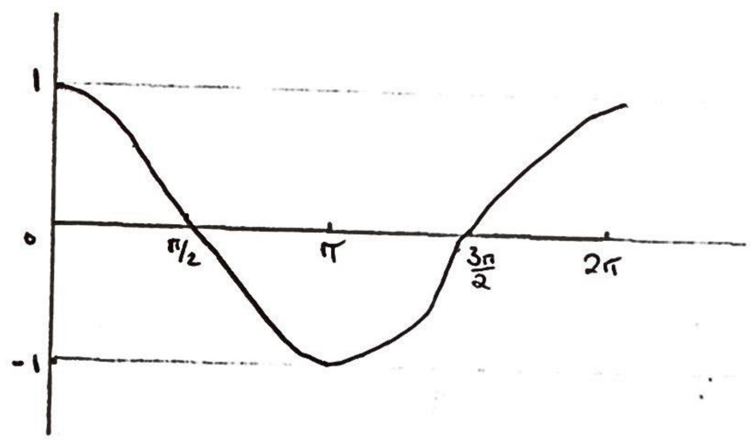
$y - 2 = -\frac{2}{3}(x-1)$

6ci. $\frac{d^2y}{dx^2} = 2 + \frac{3}{4}x^{-5/2}$

6cii. C has no maxima if $\frac{d^2y}{dx^2} > 0$ $\forall x$

since $x > 0$, $\frac{d^2y}{dx^2} > 0 \Rightarrow$ no maximum points

7a.



7bi. $\sin^2\theta = \cos\theta(2 - \cos\theta)$

$1 - \cos^2\theta = 2\cos\theta - \cos^2\theta$ $(\sin^2\theta \equiv 1 - \cos^2\theta)$

$1 = 2\cos\theta$

$\cos\theta = \frac{1}{2}$

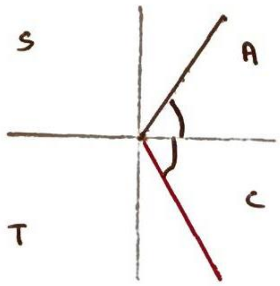
7bii. $\sin^2 2x = \cos 2x(2 - \cos 2x)$

$\cos 2x = \frac{1}{2}$ $\quad 0 \le x \le \pi$

Let $\phi = 2x$ $\quad 0 \le \phi \le 2\pi$

$\cos\phi = \frac{1}{2}$

P.V. $\phi = \frac{\pi}{3}$



$\phi = \frac{\pi}{3}, \frac{5\pi}{3}$

$2x = \frac{\pi}{3}$ $\quad x = 0.524$ (3sf)

$2x = \frac{5\pi}{3}$ $\quad x = 2.62$ (3sf)

8a. A $(0, 1)$ $\quad y = 2^{4(0)} = 1$

8b. $\int_0^1 2^{4x}\,dx$ $\quad h = \frac{1-0}{5} = 0.2$

x	y
0	1
0.2	$2^{4/5}$
0.4	$2^{8/5}$
0.6	$2^{12/5}$
0.8	$2^{16/5}$
1	2^4

$\int \approx \frac{1}{2}(0.2)\left\{(1+2^4) + 2(2^{4/5} + 2^{8/5} + 2^{12/5} + 2^{16/5})\right\}$

$= 5.54803\ldots$

$= 5.55$ (2dp)

8c. $y = 2^{4x} \rightarrow y = 2^{4x-3}$

$= 2^{4x} \cdot 2^{-3}$

$= \frac{1}{8} \cdot 2^{4x}$

$f(x) \rightarrow \frac{1}{8} f(x)$ stretch s.f. $\frac{1}{8}$ in y direction

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8d. $f(x) \rightarrow f(x-1) - \frac{1}{2}$ translation $\begin{pmatrix} 1 \\ -1/2 \end{pmatrix}$

$y = 2^{4(x-1)} - \frac{1}{2}$

at Q, $y = 0$

$0 = 2^{4(x-1)} - \frac{1}{2}$

$\frac{1}{2} = 2^{4x-4}$

$2^{-1} = 2^{4x-4}$

$-1 = 4x - 4$

$4x = 3$

$x = \frac{3}{4}$

8ei. $\log_a k = 3\log_a 2 + \log_a 5 - \log_a 4$

$= \log_a 2^3 + \log_a 5 + \log_a \frac{1}{4}$

$= \log_a (8 \times 5 \times \frac{1}{4})$

$= \log_a 10$ $\therefore$ $k = 10$

8eii. $y = \frac{5}{4}$ $y = 2^{4x-3}$

$2^{4x-3} = \frac{5}{4}$

$(4x-3)\log_{10} 2 = \log_{10} 5/4$

$4x\log_{10} 2 - 3\log_{10} 2 = \log_{10}(5/4)$

$4x\log_{10} 2 = \log_{10}(5/4) + \log_{10} 8$

$4x\log_{10} 2 = \log_{10}(8 \cdot (5/4))$

$= \log_{10} 10$

$= 1$

$x = \frac{1}{4\log_{10} 2}$

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