AQA

A Level

A Level Maths

AQA Core Maths C1 June 2010 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

AQA June 10 C1

1a.

$2x + 3y = 14$

$3y = 14 - 2x$

$y = \frac{14}{3} - \frac{2}{3}x \Rightarrow m = -\frac{2}{3}$

1bi.

D (3, 7) DC // AB $\Rightarrow$ m of DC $= -\frac{2}{3}$

$y - 7 = -\frac{2}{3}(x - 3)$

$3y - 21 = -2x + 6$

$3y = 27 - 2x$

1bii.

AD $\perp$ to AB $\Rightarrow$ m of AD $= \frac{3}{2}$

$y - 7 = \frac{3}{2}(x - 3)$

$2y - 14 = 3x - 9$

$3x - 2y + 5 = 0$

1c.

$5y - x = 6 \Rightarrow x = 5y - 6$ ①

$2x + 3y = 14$ ②

'Sub ① into ②'

$2(5y - 6) + 3y = 14$

$10y - 12 + 3y = 14$

$13y = 26$

$y = 2$

'sub $y = 2$ into ①'

$x = 5(2) - 6$

$= 4$ $\therefore$ B (4, 2)

2a.

$$(3-\sqrt{5})^2 = 9 - 6\sqrt{5} + 5$$

$$= 14 - 6\sqrt{5}$$

2b.

$$\frac{(3-\sqrt{5})^2}{1+\sqrt{5}} = \frac{14-6\sqrt{5}}{1+\sqrt{5}} \quad \times (1-\sqrt{5})$$

$$\frac{(14+6\sqrt{5})(1-\sqrt{5})}{(1+\sqrt{5})(1-\sqrt{5})} = \frac{14 - 14\sqrt{5} - 6\sqrt{5} + 6(5)}{1-5}$$

$$= \frac{4\cancel{1}4 - 20\sqrt{5}}{-4}$$

$$= -11 + 5\sqrt{5}$$

3ai.

$$p(x) = x^3 + 7x^2 + 7x - 15$$

$$p(-3) = (-3)^3 + 7(-3)^2 + 7(-3) - 15$$

$$= -27 + 63 - 21 - 15$$

$$= 0 \qquad \therefore (x+3) \text{ is a factor}$$

3aii.

$$\begin{array}{r|l}
 & x^2 + 4x - 5 \\
\hline
x+3 & x^3 + 7x^2 + 7x - 15 \\
 & x^3 + 3x^2 \downarrow \quad \downarrow \\
\hline
 & 4x^2 + 7x \\
 & 4x^2 + 12x \\
\hline
 & -5x - 15 \\
 & -5x - 15 \\
\hline
 & 0
\end{array}$$

$$p(x) = (x+3)(x^2+4x-5)$$

$$= (x+3)(x+5)(x-1)$$

3b $p(2) = (2)^3 + 7(2)^2 + 7(2) - 15$

$= 8 + 28 + 14 - 15$

$= 35 \Rightarrow$ remainder $= 35$

3ci. $p(-1) = (-1)^3 + 7(-1)^2 + 7(-1) - 15$

$= -1 + 7 - 7 - 15$

$= -16$

$p(0) = 0^3 + 7(0)^2 + 7(0) - 15$

$= -15$

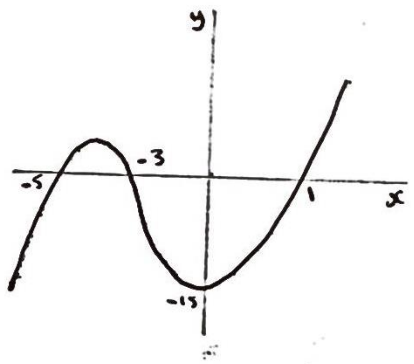
$-16 < -15 \Rightarrow p(-1) < p(0)$

3cii. $y = (x+3)(x+5)(x-1)$

roots at $1, -3, -5$

$x = 0$; $y = 3 \cdot 5 \cdot (-1) = -15$ $(0, -15)$

$+x^3 \Rightarrow$ ∿ shape



4ai. $\int_0^2 x^4 - 8x + 9 \, dx$

$= \left[\frac{1}{5}x^5 - 4x^2 + 9x \right]_0^2$

$= \frac{1}{5}(2)^5 - 4(2)^2 + 9(2) - 0$

$= \frac{32}{5} - 16 + 18$

$= \frac{42}{5}$

4aii.

A of rectangle $= 9 \times 2 = 18$

Shaded Area = Area under Curve - Rectangle .

$$= 18 - \frac{42}{5}$$

$$= \frac{90}{5} - \frac{42}{5}$$

$$= \frac{48}{5}$$

4bi.

$$y = x^4 - 8x + 9 \qquad A\ (1,2)$$

$$\frac{dy}{dx} = 4x^3 - 8$$

at A, $x = 1$, $\frac{dy}{dx} = 4(1)^3 - 8$

$$= -4$$

4bii.

$$y - 2 = -4(x - 1)$$

$$y - 2 = -4x + 4$$

$$y = -4x + 6$$

5a.

$C\ (-5, 6) \qquad r = 5$

$$(x+5)^2 + (y-6)^2 = 25$$

5bi.

$x = -2 \ , \ y = 2$

$$(-2+5)^2 + (2-6)^2 = 25$$

$$3^2 + (-4)^2 = 25$$

$$9 + 16 = 25 \ ✓$$

$\therefore$ P lies on circle

5bii. $C(-5,6) \qquad P(-2,2)$

m of $CP = \dfrac{6-2}{-5--2} = -\dfrac{4}{3}$

$CP \parallel$ normal at P, $\Rightarrow$ m of normal $= -\dfrac{4}{3}$

$y-2 = -\dfrac{4}{3}(x--2)$

$3y-6 = -4x-8$

$4x+3y+2=0$

5biii $PM = \dfrac{5}{2}$ (half the radius)

$PO = \sqrt{(2-0)^2+(2-0)^2}$

$= \sqrt{8}$

$\sqrt{8} > 5/2 \Rightarrow P$ is closer to M

6ai $SA = 2\Delta\text{'s} + 3\,\square\text{'s}$

A of $\Delta = \dfrac{1}{2}(3x \times 4x) = 6x^2$

$SA = 2(6x^2) + 3xy + 4xy + 5xy$

$= 12x^2 + 12xy$

$144 = 12x^2 + 12xy$

$xy + x^2 = 12$

6aii $V = \dfrac{1}{2}(3x \times 4x) \times y$

$= 6x^2 \cdot \dfrac{(12-x^2)}{x}$

$= 72x - 6x^3$

$xy + x^2 = 12$

$xy = 12 - x^2$

$y = \dfrac{12-x^2}{x}$

6bi.

$$V = 72x - 6x^3$$

$$\frac{dV}{dx} = 72 - 18x^2$$

6bii.

when $x = 2$, $\frac{dV}{dx} = 72 - 18(4)$

$= 0 \Rightarrow$ stat point at $x = 2$

6c.

$$\frac{d^2V}{dx^2} = -36x$$

when $x = 2$, $\frac{d^2V}{dx^2} = -36(2)$

$= -72$

$-72 < 0 \Rightarrow$ maximum when $x = 2$

7ai.

$$2x^2 - 20x + 53 = 2\left[x^2 - 10x + \frac{53}{2}\right]$$
$$= 2\left[(x-5)^2 - 25 + \frac{53}{2}\right]$$
$$= 2\left[(x-5)^2 + 3/2\right]$$
$$= 2(x-5)^2 + 3$$

7aii

$$2(x-5)^2 + 3 = 0$$
$$2(x-5)^2 = -3$$
$$(x-5)^2 = -\frac{3}{2}$$
$$x - 5 = \sqrt{-3/2} \times$$

can't $\sqrt{\text{neg.}}$ $\Rightarrow$ no real roots

7bi. $(2k-1)x^2 + (k+1)x + k = 0$

real roots $\Longleftrightarrow$ $b^2 - 4ac \geq 0$

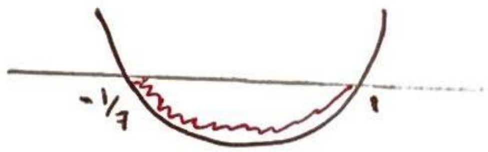
$(k+1)^2 - 4(2k-1)(k) \geq 0$

$k^2 + 2k + 1 - 8k^2 + 4k \geq 0$

$7k^2 - 6k - 1 \leq 0$

7bii. $(7k+1)(k-1) \leq 0$

C.V.s $k = 1$ or $-1/7$



$-\frac{1}{7} \leq k \leq 1$