AQA

A Level

A Level Maths

AQA Core Maths C1 January 2013 Model Solutions

Name:

M M E

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Total Marks:

AQA   Jan 13   C1

1ai.   A $(-3,2)$ B $(7,k)$ $3x + 5y = 1$

when $x = 7$, $3(7) + 5k = 1$

$21 + 5k = 1$

$5k = -20$

$k = -4$

1aii.   Mid $= \left(\dfrac{-3+7}{2}, \dfrac{2-4}{2}\right) = (2, -1)$

1b. $5y = 1 - 3x$

$y = 1 - \dfrac{3}{5}x$ $\therefore$ $m = -\dfrac{3}{5}$

1c. $\perp$ $\therefore$ $m = \dfrac{5}{3}$

$y - 2 = \dfrac{5}{3}(x - -3)$

$3y - 6 = 5x + 15$

$5x - 3y + 21 = 0$

1d. $3x + 5y = 1$ $\times 5$ $15x + 25y = 5$ ①

$5x + 8y = 4$ $\times 3$ $15x + 24y = 12$ ②

'① - ②' $y = -7$

'Sub $y = -7$ into $3x + 5y = 1$'

$3x + 5(-7) = 1$

$3x - 35 = 1$

$3x = 36$ $\therefore$ $C(12, -7)$

$x = 12$

2a.
$$y = \frac{1}{8}t^4 - t^2 + 5, \qquad 0 \le t \le 4$$

$$\frac{dy}{dt} = \frac{1}{2}t^3 - 2t$$

2bi.
$$t = 1, \quad \frac{dy}{dt} = \frac{1}{2}(1)^3 - 2(1)$$
$$= -3/2$$

2bii.
$$\frac{dy}{dx} = -3/2 < 0 \quad \Rightarrow \quad \text{decreasing}$$

2ci.
$$\frac{d^2y}{dz^2} = \frac{3}{2}t^2 - 2$$

when $t = 2$, $\quad \frac{3}{2}(2)^2 - 2 = 4$

2cii.
when $t = 2$, $\quad \frac{d^2y}{dz^2} = 4 > 0 \quad \Rightarrow \quad \text{minimum}$

3a.
$$\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$$

3aii.
$$\frac{\sqrt{8}}{\sqrt{18} + \sqrt{32}}$$

$\sqrt{8} = 2\sqrt{2}$

$\sqrt{18} = 3\sqrt{2}$

$\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$

$$= \frac{2\sqrt{2}}{3\sqrt{2} + 4\sqrt{2}}$$

$$= \frac{2\sqrt{2}}{7\sqrt{2}}$$

$$= \frac{2}{7}$$

4bii.

3b

5a.

Translation   3 left,   2 down

$p(x) = x^3 - 4x^2 - 3x + 18$

$P(-1) = (-1)^3 - 4(-1)^2 - 3(-1) + 18$

$\quad = -1 \;\; - 4 \;\; + 3 \;\; + 18$

$\quad = 16$

$\therefore$ remainder $= 16$

3

5b.

$p(3) = (3)^3 - 4(3)^2 - 3(3) + 18$

$\quad = 27 - 36 - 9 + 18$

$\quad = 54 - 54$

$\quad = 0 \qquad \therefore \;\; (x-3)$ is a factor

4a

5bi.

4a

$$\begin{array}{r} x^2 - x - 6 \\ x-3 \overline{\smash{\big)}\ x^3 - 4x^2 - 3x + 18} \\ \underline{x^3 - 3x^2} \\ -x^2 - 3x \\ \underline{-x^2 + 3x} \\ -6x + 18 \\ \underline{-6x + 18} \\ 0 \end{array}$$

4b:

$p(x) = (x-3)(x^2 - x - 6)$

$\quad = (x-3)(x-3)(x+2)$

5c.

4b

6a.
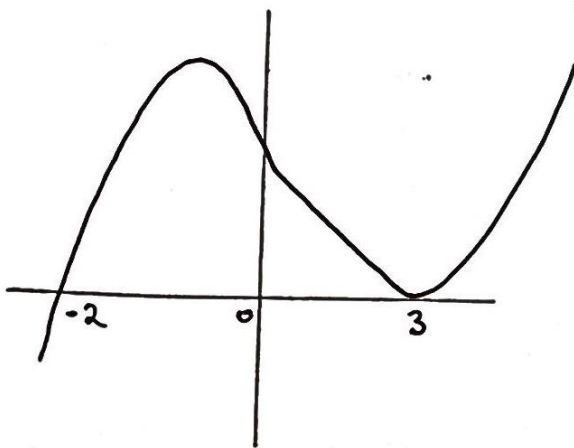
$$\frac{dy}{dx} = 10x^4 - 6x^2 + 5$$

at P, $\frac{dy}{dx} = 10(1)^4 - 6(1)^2 + 5$

$$= 10 - 6 + 5$$

$$= 9$$

$$y - 4 = 9(x - 1)$$

$$y - 4 = 9x - 9$$

$$y = 9x - 5$$

6b.

$$\int 10x^4 - 6x^2 + 5 \ \ dx$$

$$y = 2x^5 - 2x^3 + 5x + c$$

when $x = 1$, $y = 4$

$$4 = 2(1)^5 - 2(1)^3 + 5(1) + c$$

$$4 = 2 - 2 + 5 + c$$

$$c = -1$$

$$y = 2x^5 - 2x^3 + 5x - 1$$

7a.

$$x^2 + y^2 + 6x - 4y = 12$$

$x = 0$, when crosses y axis

$$y^2 - 4y = 12$$

$$y^2 - 4y - 12 = 0$$

$$(y - 6)(y + 2) = 0$$

$$y = 6 \quad \text{or} \quad y = -2$$

$$(0, 6) \quad , \quad (0, -2)$$

7b.

$$(x+3)^2 - 9 + (y-2)^2 - 4 = 12$$
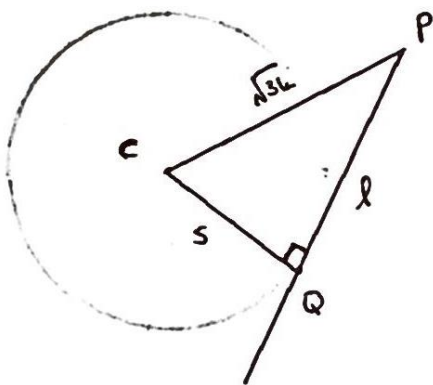
$$(x+3)^2 + (y-2)^2 = 25$$

$$r = \sqrt{25}$$

$$= 5$$

7c.

$$CP = \sqrt{(2--3)^2 + (5-2)^2}$$

$$C(-3,2) \qquad P(2,5)$$

$$= \sqrt{25 + 9}$$

$$= \sqrt{34}$$

7cii.

$$\ell^2 = (\sqrt{34})^2 - 5^2$$

$$\ell^2 = 9$$

$$\ell = 3$$

8a.

$$y = 2x^2 - x - 1 \qquad y = k(2x-3)$$

$$2x^2 - x - 1 = k(2x-3)$$

$$2x^2 - x - 1 = 2kx - 3k$$

$$2x^2 - (2k+1)x + 3k - 1 = 0$$

8bi.

2 distinct points $\Rightarrow b^2 - 4ac > 0$

$$(2k+1)^2 - 4(2)(3k-1) > 0$$

$$4k^2 + 4k + 1 - 24k + 8 > 0$$

$$4k^2 - 20k + 9 > 0$$

8b.ii.

$$4k^2 - 20k + 9 > 0$$

$$(2k - 9)(2k - 1) > 0$$

C.V.s $\quad k = \dfrac{9}{2}$ and $k = +\dfrac{1}{2}$

$$k < \dfrac{1}{2}$$

$$k > 9/2$$