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AQA

A Level

A Level Maths

AQA Core Maths C4 January 2012 Model Solutions

Name:

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Total Marks:

AQA Jan 12 C4

1a.

$$\frac{2x+3}{4x^2-1} = \frac{A}{2x-1} + \frac{B}{2x+1}$$

$$2x+3 = A(2x+1) + B(2x-1) \qquad \left((4x^2-1) = (2x+1)(2x-1)\right)$$

$x = -\frac{1}{2}$; $2 = -2B \Rightarrow B = -1$

$x = \frac{1}{2}$; $4 = 2A \Rightarrow A = 2$

$$\frac{2x+3}{4x^2-1} = \frac{2}{2x-1} - \frac{1}{2x+1}$$

1b.

$$\frac{12x^3-7x-6}{4x^2-1} = Cx + \frac{D(2x+3)}{4x^2-1}$$

$$12x^3 - 7x - 6 = Cx(4x^2-1) + D(2x+3)$$

$$= 4Cx^3 - Cx + 2Dx + 3D$$

x^3 terms : $12x^3 = 4Cx^3 \Rightarrow C = 3$

constants : $-6 = 3D \Rightarrow D = -2$

$$\frac{12x^3-7x-6}{4x^2-1} = 3x \quad \frac{-2(2x+3)}{4x^2-1}$$

1c.

$$\int_1^2 3x - \frac{2(2x+3)}{4x^2-1}\,dx$$

$$= \int_1^2 3x - 2\left(\frac{2}{2x-1} - \frac{1}{2x+1}\right) dx$$

$$= \int_1^2 3x - \frac{4}{2x-1} + \frac{2}{2x+1}\,dx$$

$$= \left[\frac{3}{2}x^2 - 2\ln|2x-1| + \ln|2x+1| \right]_1^2$$

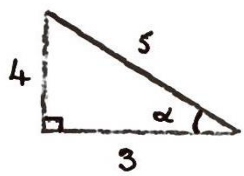
$$= \left(\frac{3}{2}(2)^2 - 2\ln 3 + \ln 5 \right) - \left(\frac{3}{2}(1)^2 - 2\ln 1 + \ln 3 \right)$$

$$= 6 - 2\ln 3 + \ln 5 - \frac{3}{2} - \ln 3$$

$$= \frac{9}{2} - 3\ln 3 + \ln 5$$

$$= \frac{9}{2} + \ln\left(\frac{5}{27}\right) \qquad (3\ln 3 = \ln 3^3 = \ln 27)$$

2ai. $\cos\alpha = \frac{3}{5}$



$\cos\alpha = \frac{adj}{hyp} = \frac{3}{5}$

$\tan\alpha = \frac{opp}{adj} = \frac{4}{3}$

2aii.

$$\sin\beta = \frac{1}{2}$$

$$\operatorname{cosec}\beta = 2$$

$$\operatorname{cosec}^2\beta = 4$$

$$1 + \cot^2\beta = 4 \qquad (1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta)$$

$$\cot^2\beta = 3$$

$$\tan^2\beta = \frac{1}{3}$$

$$\tan\beta = \pm\frac{1}{\sqrt{3}}$$

$$\therefore \tan\beta = -\frac{1}{\sqrt{3}} \quad \text{since } \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ < 90^\circ$$

2b.

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$

$$= \frac{4/3 + \left(-\frac{1}{\sqrt{3}}\right)}{1-(4/3)\left(-\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{\frac{4-\sqrt{3}}{3}}{\frac{9+4\sqrt{3}}{9}} \qquad \times 9$$

$$= \frac{12-3\sqrt{3}}{9+4\sqrt{3}}$$

__NB__ answer different to mark scheme - but still correct awarded full marks as 'any equivalent form' under 'Special Case' - special case answer equivalent if numerator and denominator are multiplied by -1.

3a.

$$(1+6x)^{2/3} \approx 1 + (2/3)(6x) + \frac{(2/3)(-1/3)}{2}(6x)^2 + \ldots$$

$$= 1 + 4x - 4x^2 + \ldots$$

3b.

$$(8+6x)^{2/3} = \left[8\left(1+3/4x\right)\right]^{2/3}$$

$$= 8^{2/3}\left(1+3/4x\right)^{2/3}$$

$$= 4\left(1 + (2/3)(3/4x) + \frac{(2/3)(-1/3)}{2}(3/4x)^2 + \ldots\right)$$

$$= 4\left(1 + \frac{1}{2}x - \frac{1}{16}x^2 + \ldots\right)$$

$$= 4 + 2x - \frac{1}{4}x^2 + \ldots$$

3e. $(8+6x)^{2/3} = 100^{1/3}$ (cube both sides)

$(8+6x)^2 = 100$

$8+6x = 10$

$x = 1/3$

so $(8+6x)^{2/3} = 100^{1/3}$ when $x = \frac{1}{3}$

so $\sqrt[3]{100} \approx 4 + 2\left(\frac{1}{3}\right) - \frac{1}{4}\left(\frac{1}{3}\right)^2 + \dots$

$= \frac{167}{36}$

4a. $P = Ae^{0.125t}$

when $t=0$, $P = 500$

$500 = Ae^0 \Rightarrow A = 500$

$P = 500e^{0.125t}$

$t = 60$; $P = 500e^{60 \times 0.125}$

$= 904{,}021 \dots$

$= 904{,}000$ (nearest 1000)

4bi. $Q = 500{,}000e^{-0.125t}$, $P = 500e^{0.125t}$

$500{,}000e^{-0.125t} = 500e^{0.125t}$, $\times e^{0.125t}$

$500{,}000 = 500e^{0.25t}$

$e^{0.25t} = 1000$

$0.25t = \ln 1000$

$t = \frac{\ln 1000}{0.25} = 27.6$ (1 dp)

4bii. at $t = T$, $P - Q = 45{,}000$

$$500e^{0.125T} - 500{,}000e^{-0.125T} = 45{,}000 \qquad (\div 500)$$

$$e^{0.125T} - 1000e^{-0.125T} - 90 = 0 \qquad (\times e^{0.125T})$$

$$\left(e^{0.125T}\right)^2 - 1000 - 90e^{0.125T} = 0$$

$$\left(e^{\frac{1}{8}T}\right)^2 - 90e^{\frac{1}{8}T} - 1000 = 0$$

let $x = e^{\frac{1}{8}T}$:

$$x^2 - 90x - 1000 = 0$$

$$(x - 100)(x + 10) = 0$$

$$x = 100 \text{ or } -10$$

$$e^{\frac{1}{8}T} = 100$$

$e^{\frac{1}{8}T} \neq -10$ since $e^x > 0 \ \forall x$

$$\frac{1}{8}T = \ln 100$$

$$T = 8\ln 100$$

$$= 36.8 \quad (1\text{d.p.})$$

5a. $x = 8t^2 - t$

$$y = \frac{3}{t}$$

$$t = \frac{3}{y}$$

$$x = 8\left(\frac{3}{y}\right)^2 - \frac{3}{y}$$

$$x = \frac{72}{y^2} - \frac{3}{y}$$

$$xy^2 = 72 - 3y$$

$$xy^2 + 3y = 72$$

5bi.

$x = 8t^2 - t \qquad y = 3t^{-1}$

$\frac{dx}{dt} = 16t - 1 \qquad \frac{dy}{dt} = -3t^{-2}$

$\frac{dy}{dx} = \frac{-3t^{-2}}{16t-1}$

when $t = \frac{1}{4}$; $\frac{dy}{dx} = \frac{-3(0.25)^{-2}}{16(0.25)-1} = -16$

$x = 8(0.25)^2 - 0.25 = 1/4$

$y = 3(0.25)^{-1} = 12$

$(1/4, 12)$

$y - 12 = -16(x - 1/4)$

$y - 12 = -16x + 4$

$y = -16x + 16$

5bii.

when $x = 3/2$, $y = -16(3/2) + 16$

$= -8$

verify in $xy^2 + 3y = 72$

$(3/2)(-8)^2 + 3(-8) = 72$

$96 - 24 = 72$ ✓

$\therefore$ lines intersect when $x = 3/2$

6a

let $f(x) = 16x^3 + 11x - 15$

$f(3/4) = 16(3/4)^3 + 11(3/4) - 15$

$16\left(\frac{27}{64}\right) + 11\left(\frac{3}{4}\right) - 15 = 0$

$\therefore$ $(4x - 3)$ is a factor

6b.

$$27\cos\theta\cos 2\theta + 19\sin\theta\sin 2\theta - 15 = 0 \qquad x = \cos\theta$$

$$27\cos\theta(2\cos^2\theta - 1) + 19\sin\theta . 2\sin\theta\cos\theta - 15 = 0$$

$$54\cos^3\theta - 27\cos\theta + 38\sin^2\theta\cos\theta - 15 = 0$$

$$54\cos^3\theta - 27\cos\theta + 38(1 - \cos^2\theta)\cos\theta - 15 = 0$$

$$54\cos^3\theta - 27\cos\theta + 38\cos\theta - 38\cos^3\theta - 15 = 0$$

$$16\cos^3\theta + 11\cos\theta - 15 = 0$$

$$16x^3 + 11x - 15 = 0$$

6c.

$$
\begin{array}{r|l}
 & 4x^2 + 3x + 5 \\
\hline
4x - 3 & 16x^3 + 0x^2 + 11x - 15 \\
 & \underline{16x^3 - 12x^2} \\
 & 12x^2 + 11x \\
 & \underline{12x^2 - 9x} \\
 & 20x - 15 \\
 & \underline{20x - 15} \\
 & 0
\end{array}
$$

$$f(x) = (4x^2 + 3x + 5)(4x - 3)$$

either $4x - 3 = 0$

$4\cos\theta = 3$

$\cos\theta = \frac{3}{4}$

or $4x^2 + 3x + 5 = 0$

disc $= (+3)^2 - 4(4)(5)$

$= -71$

disc. < 0 $\therefore$ no real solutions

$\therefore \cos\theta = \frac{3}{4}$ only solution

7.

$$\frac{dy}{dx} = y^2 x \sin 3x$$

$$\int y^{-2}\, dy = \int x \sin 3x \, dx$$

$$\int x \sin 3x \, dx \qquad \text{Parts} \qquad u = x \quad v' = \sin 3x$$

$$u' = 1 \qquad v = -\frac{1}{3}\cos 3x$$

$$= -\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x \, dx$$

$$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + c$$

$$\int y^{-2}\, dy = -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + c$$

$$-y^{-1} = -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + c$$

when $y = 1, \; x = \pi/6$

$$-1 = -\frac{1}{3}\left(\frac{\pi}{6}\right)\cos\left(\pi/2\right) + \frac{1}{9}\sin\left(\pi/2\right) + c$$

$$-1 = 0 + \frac{1}{9} + c \qquad c = -\frac{10}{9}$$

$$-\frac{1}{y} = -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x - \frac{10}{9}$$

$$-\frac{1}{y} = \frac{-3x\cos 3x}{9} + \frac{\sin 3x}{9} - \frac{10}{9}$$

$$-\frac{1}{y} = \frac{-3x\cos 3x + \sin 3x - 10}{9}$$

$$y = \frac{9}{3x\cos 3x - \sin 3x + 10}$$

8ai. $A\ (4, -2, 3) \qquad B\ (2, 0, -1)$

$l: \underline{r} = \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 2 \\ -4 \end{pmatrix}$

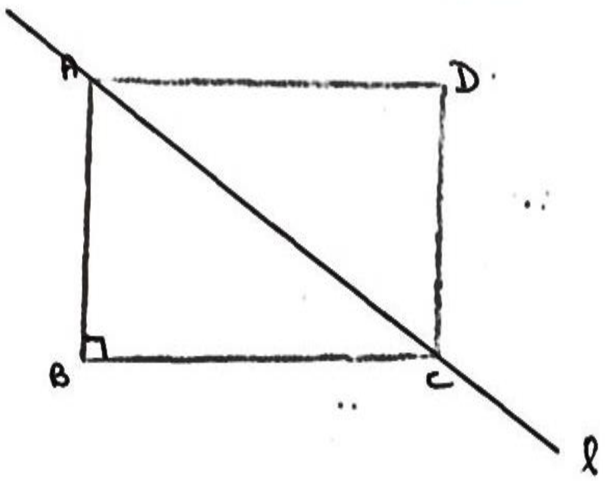
8aii. $\cos\theta = \dfrac{\begin{pmatrix} -2 \\ 2 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix}}{\sqrt{2^2+2^2+4^2} \cdot \sqrt{1^2+5^2+2^2}}$

$= \dfrac{-2(1) + 2(5) - 4(-2)}{\sqrt{24} \cdot \sqrt{30}}$

$\cos\theta = \dfrac{16}{12\sqrt{5}}$

$\theta = 53°$ (nearest degree)

8b.



$\overrightarrow{AB} \perp \overrightarrow{BC}$

$\therefore \quad \overrightarrow{AB} \cdot \overrightarrow{BC} = 0$

C lies on l so $\overrightarrow{OC} = \begin{pmatrix} 4+\lambda \\ -2+5\lambda \\ 3-2\lambda \end{pmatrix}$ for some λ

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$$

$$= \begin{pmatrix} 4+\lambda \\ -2+5\lambda \\ 3-2\lambda \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2+\lambda \\ -2+5\lambda \\ 4-2\lambda \end{pmatrix}$$

$$\overrightarrow{AB} \cdot \overrightarrow{BC} = 0$$

$$\begin{pmatrix} -2 \\ 2 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 2+\lambda \\ -2+5\lambda \\ 4-2\lambda \end{pmatrix} = 0$$

$$-2(2+\lambda) + 2(-2+5\lambda) - 4(4-2\lambda) = 0$$

$$-4 - 2\lambda - 4 + 10\lambda - 16 + 8\lambda = 0$$

$$16\lambda = 24$$

$$\lambda = 3/2$$

so $\overrightarrow{OC} = \begin{pmatrix} 4+3/2 \\ -2+15/2 \\ 3-6/2 \end{pmatrix} = \begin{pmatrix} 11/2 \\ 11/2 \\ 0 \end{pmatrix}$ $\quad C\,(5.5, 5.5, 0)$

$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC}$ $\quad$ ($\overrightarrow{BC}$ parallel to $\overrightarrow{AD}$)

$$\overrightarrow{BC} = \begin{pmatrix} 2+3/2 \\ -2+15/2 \\ 3-6/2 \end{pmatrix} = \begin{pmatrix} 3.5 \\ 5.5 \\ 1 \end{pmatrix}$$

$$\overrightarrow{OD} = \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 3.5 \\ 5.5 \\ 1 \end{pmatrix} = \begin{pmatrix} 7.5 \\ 3.5 \\ 4 \end{pmatrix}$$

$$D\,(7.5, 3.5, 4)$$