AQA

A Level

A Level Maths

AQA Core Maths C3 January 2012 Model Solutions

Name:

M M E

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Total Marks:

AQA Jan 12 C3

1a.

$\int_0^3 4^x \, dx$ $h = \frac{3-0}{6} = 0.5$

x	y
0	1
0.5	2
1	4
1.5	8
2	16
2.5	32
3	64

$\int \approx \frac{1}{3}(0.5)\left\{(1+64) + 4(2+8+32) + 2(4+16)\right\}$

$= 45.5$

1bi.

$y = 4^x$, $y = 8-2x$

intersect when $4^x = 8-2x$

let $f(x) = 4^x - 8 + 2x$

$f(1.2) = -0.32196...$

$f(1.3) = 0.66286...$

change of sign $\therefore$ $1.2 < \alpha < 1.3$

1bii.

$x_{n+1} = \frac{\ln(8-2x_n)}{\ln 4}$

$x_1 = 1.2$

$x_2 = 1.243$

$x_3 = 1.232$

2a.

$f(x) = \frac{63}{4x-1}$ $1 \leq x \leq 16$

$f(1) = \frac{63}{3} = 21$

$f(16) = \frac{63}{63} = 1$ $\therefore$ $1 \leq f(x) \leq 16$

2bi. let $y = \frac{63}{4x-1}$

$4xy - y = 63$

$4xy = 63 + y$

$x = \frac{63+y}{4y}$

$\therefore \quad f^{-1}(x) = \frac{63+x}{4x}$

2bii. $f^{-1}(x) = 1 \quad \Rightarrow \quad \frac{63+x}{4x} = 1$

$63 + x = 4x$

$63 = 3x$

$x = 21$

2ci. $g(x) = x^2 \qquad -4 \leq x \leq -1$

$fg(x) = f(x^2)$

$= \frac{63}{4x^2-1}$

2cii. $fg(x) = 1 \quad \Rightarrow \quad \frac{63}{4x^2-1} = 1$

$63 = 4x^2 - 1$

$64 = 4x^2$

$x^2 = 16$

$x = \pm 4$

but $-4 \leq x \leq -1$

$\therefore \quad x = -4$

3a.

$$y = 4x^3 - 6x + 1$$

$$\frac{dy}{dx} = 12x^2 - 6$$

3b.

$$\int_2^3 \frac{2x^2 - 1}{4x^3 - 6x + 1} \, dx$$

$$= \frac{1}{6} \int_2^3 \frac{12x^2 - 6}{4x^3 - 6x + 1} \, dx$$

$$= \frac{1}{6} \left[\ln|4x^3 - 6x + 1| \right]_2^3$$

$$= \frac{1}{6} \left(\ln|4(3)^3 - 6(3) + 1| - \ln|4(2)^3 - 6(2) + 1| \right)$$

$$= \frac{1}{6} (\ln 91 - \ln 21)$$

$$= \frac{1}{6} \ln \frac{91}{21}$$

$$= \frac{1}{6} \ln \frac{13}{3}$$

4a.

$$\tan^2\theta = 3(3 - \sec\theta) \qquad 0 < \theta < 360$$

$$\sec^2\theta - 1 = 9 - 3\sec\theta \qquad (\tan^2\theta \equiv \sec^2\theta - 1)$$

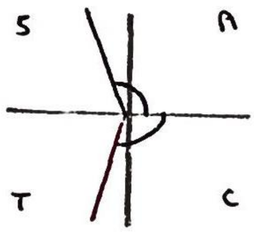
$$\sec^2\theta + 3\sec\theta - 10 = 0$$

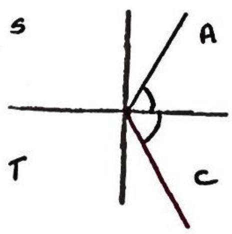
$$(\sec\theta + 5)(\sec\theta - 2) = 0$$

$\sec\theta = -5$ or $\sec\theta = 2$

$\cos\theta = -1/5$ $\qquad$ $\cos\theta = 1/2$

P.V. $101.54°$ $\qquad$ P.V. $60°$





$\theta = 101.5°, 258.5°$ $\qquad$ $\theta = 60°, 300°$

4b. $\tan^2(4x-10) = 3(3 - \sec(4x-10))$

$4x - 10 = \theta$

$4x - 10 = 60° \Rightarrow x = 17.5°$

$= 101.537° \Rightarrow x = 27.9°$

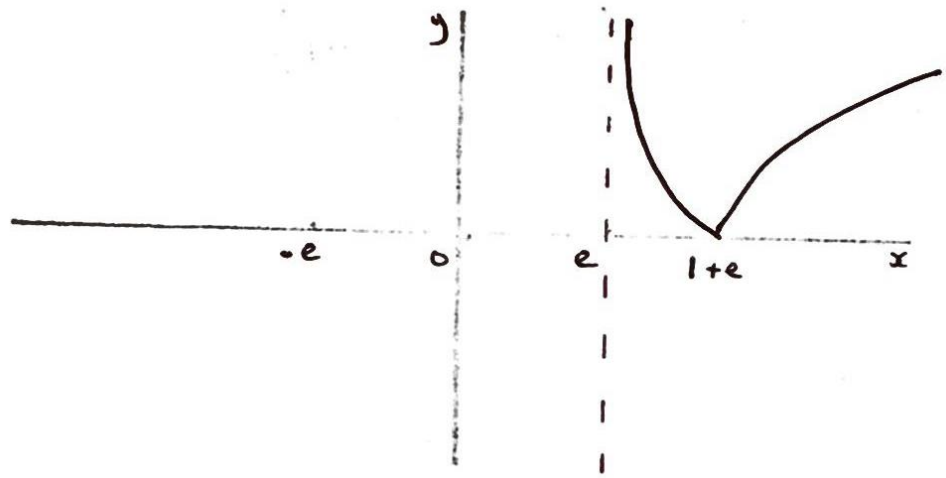
$= 258.463 \Rightarrow x = 67.1°$

$= 300° \Rightarrow x = 77.5°$

5a. $y = \ln x \rightarrow y = 4\ln x$ 'stretch s.f. 4 in y direction'

$y = 4\ln x \rightarrow y = 4\ln(x-e)$ 'translation $\begin{pmatrix} e \\ 0 \end{pmatrix}$'

5b.



5ci. $|4\ln(x-e)| = 4$

$4\ln(x-e) = 4$ or $4\ln(x-e) = -4$

$\ln(x-e) = 1$ $\qquad$ $\ln(x-e) = -1$

$x - e = e^1$ $\qquad$ $x - e = e^{-1}$

$x = 2e$ $\qquad$ $x = e + \frac{1}{e}$

5cii. $e < x \leq e + \frac{1}{e}$

6a. $x = \dfrac{1}{\sin\theta}$ Quotient: $f = 1$, $f' = 0$, $g = \sin\theta$, $g' = \cos\theta$

$$\frac{dx}{d\theta} = \frac{0 - \cos\theta}{\sin^2\theta}$$

$$= \frac{-\cos\theta}{\sin^2\theta} = -\frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\sin\theta}$$

$$= -\operatorname{cosec}\theta\cot\theta$$

6b. $$\int_{\sqrt{2}}^{2} \frac{1}{x^2\sqrt{x^2-1}}\,dx$$

$x = \operatorname{cosec}\theta$

$dx = -\operatorname{cosec}\theta\cot\theta\,d\theta$

$x^2 = \operatorname{cosec}^2\theta$

x	2	$\sqrt{2}$
θ	$\pi/6$	$\pi/4$

$2 = \operatorname{cosec}\theta \rightarrow \theta = \sin^{-1}\left(\frac{1}{2}\right) = \pi/6$

$\sqrt{2} = \operatorname{cosec}\theta \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \pi/4$

$$\int_{\pi/4}^{\pi/6} \frac{1}{\operatorname{cosec}^2\theta\sqrt{\operatorname{cosec}^2\theta-1}}\cdot -\operatorname{cosec}\theta\cot\theta\,d\theta$$

$$\int_{\pi/4}^{\pi/6} \frac{-\cot\theta}{\operatorname{cosec}\theta\sqrt{\operatorname{cosec}^2\theta-1}}\,d\theta$$

$$\int_{\pi/4}^{\pi/6} -\frac{\cot\theta}{\operatorname{cosec}\theta\sqrt{\cot^2\theta}}\,d\theta$$

$$\int_{\pi/4}^{\pi/6} -\frac{1}{\operatorname{cosec}\theta}\,d\theta = \int_{\pi/4}^{\pi/6} -\sin\theta\,d\theta$$

$$= \Big[\cos\theta\Big]_{\pi/4}^{\pi/6}$$

$$= \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}$$

$$= 0.159 \quad (3\text{s.f.})$$

7a.

$y = x^2 e^{-\frac{x}{4}}$

$\frac{dy}{dx} = 2xe^{-\frac{x}{4}} + x^2 \cdot \left(-\frac{1}{4} e^{-\frac{x}{4}}\right)$

$= 2xe^{-\frac{x}{4}} - \frac{x^2}{4} e^{-\frac{x}{4}}$

at stat. pt. $\frac{dy}{dx} = 0$

$2xe^{-\frac{x}{4}} - \frac{x^2}{4} e^{-\frac{x}{4}} = 0$ ($e^{-x/4} > 0$ so can divide by it)

$2x - \frac{x^2}{4} = 0$

$x^2 - 8x = 0$

$x(x-8) = 0$

$x = 0$ or $x = 8$

when $x = 0$, $y = 0^2 e^{-\frac{0}{4}}$
$= 0$ $(0, 0)$

when $x = 8$, $y = 8^2 e^{-\frac{8}{4}}$
$= 64e^{-2}$ $(8, 64e^{-2})$

7b.

$\int_0^4 x^2 e^{-x/4}\, dx$ Parts : $u = x^2$ $v' = e^{-x/4}$
$u' = 2x$ $v = -4e^{-x/4}$

$= -4x^2 e^{-x/4} - \int -8xe^{-x/4}\, dx$

$= -4x^2 e^{-x/4} + \int 8xe^{-x/4}\, dx$ $(*)$

$\int 8xe^{-x/4}\,dx$ Parts: $u = 8x$ $v' = e^{-x/4}$

$u' = 8$ $v = -4e^{-x/4}$

$= -32xe^{-x/4} - \int -32e^{-x/4}\,dx$

$= -32xe^{-x/4} + \int 32e^{-x/4}\,dx$

$= -32xe^{-x/4} - 128e^{-x/4}$ 'sub back into $(*)$'

$\int_0^4 x^2e^{-x/4}\,dx = \left[-4x^2e^{-x/4} + -32xe^{-x/4} - 128e^{-x/4}\right]_0^4$

$= \left(-4(4)^2e^{-4/4} - 32(4)e^{-4/4} - 128e^{-4/4}\right) - \left(-128e^{-0}\right)$

$= -64e^{-1} - 128e^{-1} - 128e^{-1} - (-128)$

$= 128 - 320e^{-1}$

7bii. $V = \pi\int_0^4 y^2\,dx$

$y = 3xe^{-x/8}$

$y^2 = 9x^2e^{-x/4}$

$V = \pi\int_0^4 9x^2e^{-x/4}\,dx$

$= 9\pi\int_0^4 x^2e^{-x/4}\,dx$

$= 9\pi\left(128 - 320e^{-1}\right)$