AQA

A Level

A Level Maths

AQA Core Maths C2 January 2012 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

AQA Jan 12 C2

1a. $A = \frac{1}{2}r^2\theta$

$21.6 = \frac{1}{2}(6^2)\theta$

$\theta = \frac{21.6}{18}$

$= 1.2$

1b. $l = r\theta$

$= 6(1.2)$

$= 7.2$

2a. $\int_0^4 \frac{2^x}{x+1}\,dx$ $h = \frac{4-0}{4} = 1$

x	y
0	1
1	1
2	4/3
3	8/4
4	16/5

$\int \approx \frac{1}{2}(1)\left\{(1+16/5) + 2\left(1 + 4/3 + 8/4\right)\right\}$

$= 6.43$ (3sf)

2b. more strips

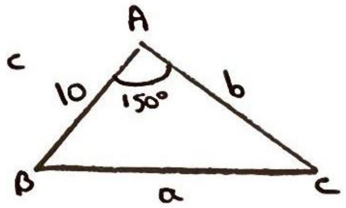
3a. $\sqrt[4]{x^3} = (x^3)^{1/4} = x^{3/4}$

3b. $\frac{1-x^2}{\sqrt[4]{x^3}} = \frac{1-x^2}{x^{3/4}}$

$= \frac{1}{x^{3/4}} - \frac{x^2}{x^{3/4}}$

$= x^{-3/4} - x^{5/4}$

4a.i



$$\text{Area} = \frac{1}{2} bc \sin A$$

$$40 = \frac{1}{2} b . 10 . \sin 150°$$

$$40 = b \times 5 \sin 150$$

$$b = \frac{40}{5 \sin 150°}$$

$$= 16$$

4b.

BC = a

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 16^2 + 10^2 - 2(16)(10) \cos 150°$$

$$a = \sqrt{633.128129...}$$

$$= 25.16 \quad (2dp)$$

4c.

$$\frac{\sin C}{10} = \frac{\sin 150}{16}$$

$$C = \sin^{-1}\left(\frac{10 \sin 150}{16}\right)$$

$$= 11.5° \quad (1dp)$$

$$B = 180 - 150 - 11.5$$

$$= 18.5°$$

$\therefore$ smallest angle C = 11.5°

5a.i.

$$F(x) = \left(1 + \frac{x}{3}\right)^6$$

$$F(6x) = (1 + 2x)^6$$

$\therefore$ stretch s.f. $\frac{1}{6}$ in x direction

5ai. Translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ $\quad f(x) \rightarrow f(x-3)$

$$f(x) = \left(1 + \frac{x}{3}\right)^6$$

$$f(x-3) = \left(1 + \frac{x-3}{3}\right)^6$$

$$= \left(1 + \frac{x}{3} - 1\right)^6$$

$$= \left(\frac{x}{3}\right)^6 = g(x)$$

5b. $$\left(1 + \frac{x}{3}\right)^6 = 1^6 + {}^6C_1 1^5\left(\frac{x}{3}\right) + {}^6C_2 1^4 \left(\frac{x}{3}\right)^2 + {}^6C_3 1^3 \left(\frac{x}{3}\right)^3 + \dots$$

$$= 1 + 2x + \frac{5}{3}x^2 + \frac{20}{27}x^3 + \dots$$

$a = 2$, $b = 5/3$, $c = \frac{20}{27}$

6a. AP. $S_{25} = 3500$

$$S_{25} = \frac{25}{2}\left(2a + (25-1)d\right) = 3500 \quad (\times 2/25)$$

$$2a + 24d = 280 \quad (\div 2)$$

$$a + 12d = 140 \quad ①$$

6b. $$U_5 = a + 4d = 100 \quad ②$$

'① − ②'

$$8d = 40$$

$$d = 5$$

'sub in ②'

$$a + 4(5) = 100$$

$$a = 80$$

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6c.

$$33\left(\sum_{n=1}^{25} u_n - \sum_{n=1}^{k} u_n\right) = 67\sum_{n=1}^{k} u_n$$

$$\sum_{n=1}^{25} u_n = 3500$$

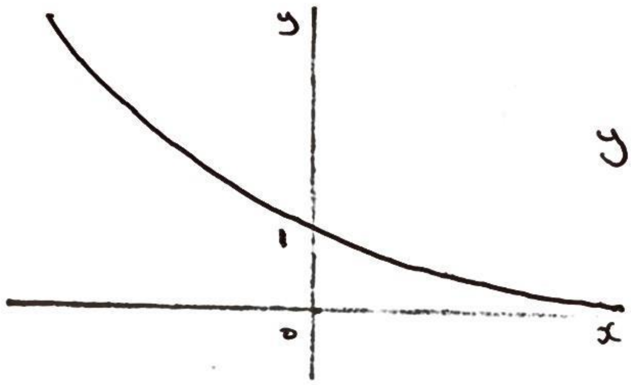
$$33 \times 3500 - 33\sum_{n=1}^{k} u_n = 67\sum_{n=1}^{k} u_n$$

$$115{,}500 = 100\sum_{n=1}^{k} u_n$$

$$\sum_{n=1}^{k} u_n = \frac{115{,}500}{100}$$

$$= 1{,}155$$

7a.



$$y = \frac{1}{2^x}$$

7b.

$$\frac{1}{2^x} = \frac{5}{4}$$

$$2^x = \frac{4}{5}$$

$$\log 2^x = \log(4/5)$$

$$x\log 2 = \log(4/5)$$

$$x = \frac{\log(4/5)}{\log 2}$$

$$= -0.322 \quad (3sf)$$

7c.

$$\log_a b^2 + 3\log_a y = 3 + 2\log_a\left(\frac{y}{a}\right)$$

$$\log_a b^2 + 3\log_a y = 3 + 2\log_a y - 2\log_a a \qquad (-2\log_a y)$$

$$\log_a b^2 + \log_a y = 3 - 2\log_a a \qquad (\log_a a = 1)$$

$$\log_a b^2 y = 3 - 2(1)$$

$$\log_a b^2 y = 1$$

$$b^2 y = a^1$$

$$y = \frac{a}{b^2}$$

8a.

$$2\sin\theta = 7\cos\theta \qquad (\div \cos\theta)$$

$$2\frac{\sin\theta}{\cos\theta} = 7$$

$$2\tan\theta = 7 \qquad \left(\tan\theta = \frac{\sin\theta}{\cos\theta}\right)$$

$$\tan\theta = \frac{7}{2}$$

8bi.

$$6\sin^2 x = 4 + \cos x$$

$$6(1-\cos^2 x) = 4 + \cos x \qquad (\sin^2\theta \equiv 1 - \cos^2\theta \ \forall\ \theta \in \mathbb{R})$$

$$6 - 6\cos^2 x = 4 + \cos x$$

$$6\cos^2 x + \cos x - 2 = 0$$

8bii.

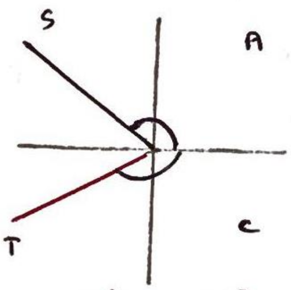
$$(3\cos x + 2)(2\cos x - 1) = 0$$

$$0 < x < 360$$

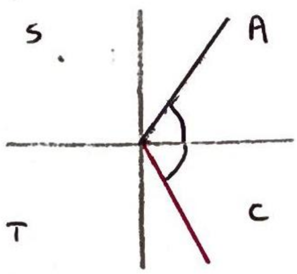
$\cos x = -2/3$ or $\cos x = \frac{1}{2}$

P.V. $x = 131.81°$

P.V. $x = 60°$



$x = 132°, \ 228°$



$x = 60°, 300°$

9a. $y = 12x - 3x^{5/3}$

$\frac{dy}{dx} = 12 - 5x^{2/3}$

9bi. when $x = 0$, $\frac{dy}{dx} = 12 - 5(0)^{2/3}$ $(0,0)$

$= 12$

$y - 0 = 12(x - 0)$

$y = 12x$

9bii. when $x = 8$, $\frac{dy}{dx} = 12 - 5(8)^{2/3}$ $(8,0)$

$= -8$

$y - 0 = -8(x - 8)$

$y = -8x + 64$

$y + 8x = 64$

9c. $\int 12x - 3x^{5/3}\,dx = 6x^2 - \frac{9}{8}x^{8/3} + c$

9d Shaded Area = Area of $\triangle$ OPA - Area under curve

Need P - point of intersection of $y = 12x$ and $y + 8x = 64$

sub $y = 12x$ into $y + 8x = 64$

$20x = 64$

$x = 3.2$

$y = 12(3.2) = 38.4$

Area of $\triangle$OPA $= \frac{1}{2}(b \times h)$ (h = y coordinate of P)

$= \frac{1}{2}(8 \times 38.4)$

$= 153.6$

Area under curve

$= \left[6x^2 - \frac{9}{8}x^{8/3} \right]_0^8$

$= \left(6(8)^2 - \frac{9}{8}(8)^{8/3} \right) - 0$

$= 96$

$\therefore$ Shaded Area $= 153.6 - 96$

$= 57.6$