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AQA

A Level

A Level Maths

AQA Core Maths C3 January 2011 Model Solutions

Name:

M M E

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Total Marks:

AQA Jan 11 C3

1a. $y = (x^3 - 1)^6$

$\frac{dy}{dx} = 6.\ 3x^2 . (x^3-1)^5$

$= 18x^2(x^3-1)^5$

1bi. $y = x \ln x$

$\frac{dy}{dx} = \ln x + x . \frac{1}{x}$

$= \ln x + 1$

1bii. when $x = e$, $y = e \ln e = e$ (e, e)

$\frac{dy}{dx} = \ln e + 1 = 2$

$y - e = 2(x - e)$

$y = 2x - e$

2a. $y = (x^2-4)\ln(x+2)$, $y = 15$

intersect when $(x^2-4)\ln(x+2) = 15$

let $f(x) = (x^2-4)\ln(x+2) - 15$

$f(3.5) = -0.9358\ldots$

$f(3.6) = 0.43598\ldots$

change of sign $\Rightarrow$ $3.5 < \alpha < 3.6$

2b. $(x^2-4)\ln(x+2) = 15$ $(\div \ln(x+2))$

$(x^2-4) = \frac{15}{\ln(x+2)}$ $(+4)$

$x^2 = 4 + \frac{15}{\ln(x+2)}$

$x = \pm\sqrt{4 + \frac{15}{\ln(x+2)}}$

2c.

$$x_{n+1} = \sqrt{4 + \frac{15}{\ln(x_n + 2)}}$$

$x_1 = 3.5$

$x_2 = 3.578$

$x_3 = 3.568$ (3dp)

3ai.

$x = \tan(3y + 1)$

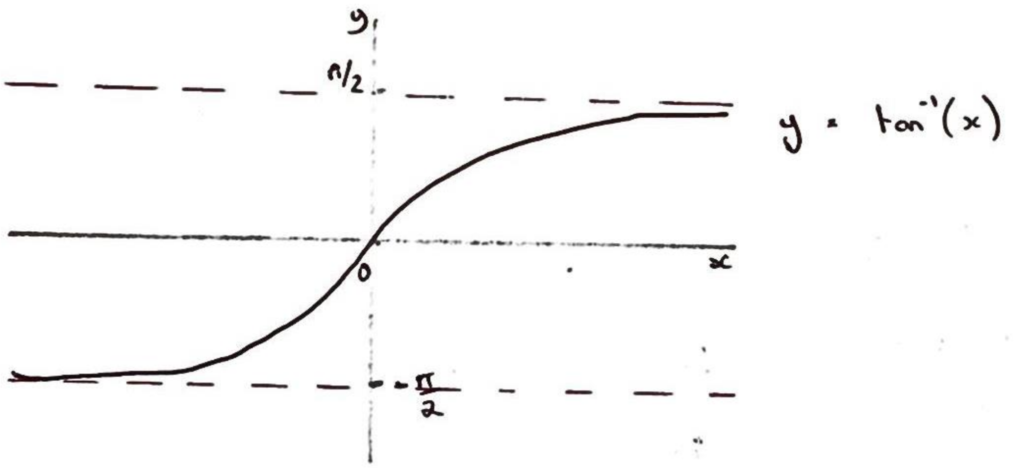
$$\frac{dx}{dy} = 3\sec^2(3y + 1)$$

3aii.

when $y = -\frac{1}{3}$, $\frac{dx}{dy} = 3\sec^2\left(3(-1/3) + 1\right) = 3$

$\therefore \frac{dy}{dx} = \frac{1}{3}$

3b.



4a.

$f(x) = 3\cos\frac{1}{2}x$ $\quad 0 \le x \le 2\pi$

$g(x) = |x|$ $\quad \forall x \in \mathbb{R}$

$f(0) = 3\cos 0 = 3$

$f(2\pi) = 3\cos\pi = -3$

$-3 \le f(x) \le 3$

4bi. Let $y = 3\cos\left(\frac{1}{2}x\right)$

$$\frac{y}{3} = \cos\left(\frac{1}{2}x\right)$$

$$\cos^{-1}\left(\frac{y}{3}\right) = \frac{1}{2}x$$

$$x = 2\cos^{-1}\left(\frac{y}{3}\right)$$

$$\therefore \quad f^{-1}(x) = 2\cos^{-1}\left(\frac{x}{3}\right)$$

4bii.

$$f^{-1}(x) = 1$$

$$2\cos^{-1}\left(\frac{x}{3}\right) = 1$$

$$\cos^{-1}\left(\frac{x}{3}\right) = 1/2$$

$$\frac{x}{3} = \cos(1/2)$$

$$x = 3\cos(1/2)$$

4ci.

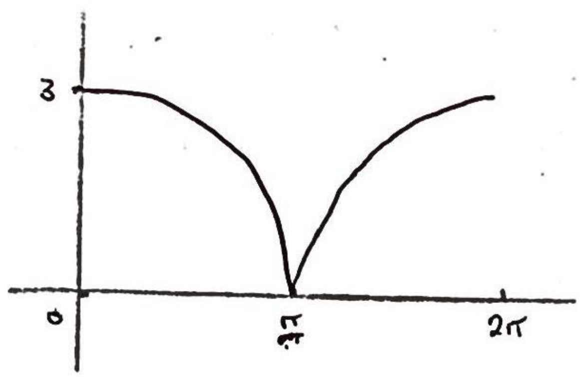
$$gf(x) = g\left(3\cos\tfrac{1}{2}x\right)$$

$$= \left|3\cos\tfrac{1}{2}x\right|$$

4cii.

$$y = \left|3\cos\tfrac{1}{2}x\right|$$

$0 \le x \le 2\pi$



4d.

$y = \cos x \rightarrow y = 3\cos x$ stretch s.f. 3 in y direction

$y = 3\cos x \rightarrow y = 3\cos\frac{1}{2}x$ stretch s.f. 2 in x direction

5a. $\int \frac{1}{3+2x} \, dx$

$= \frac{1}{2} \int \frac{2}{3+2x} \, dx$

$= \frac{1}{2} \ln|3+2x| + c$

5b. $\int x \sin\left(\frac{x}{2}\right) dx$ Parts

$u = x$ $\quad v' = \sin\left(\frac{x}{2}\right)$

$u' = 1$ $\quad v = -2\cos\left(\frac{x}{2}\right)$

$= -2x\cos\left(\frac{x}{2}\right) - \int -2\cos\left(\frac{x}{2}\right) . 1 \, dx$

$= -2x\cos\left(\frac{x}{2}\right) + \int 2\cos\left(\frac{x}{2}\right) dx$

$= -2x\cos\left(\frac{x}{2}\right) + 4\sin\left(\frac{x}{2}\right) + c$

6a. $\int_0^{0.4} \cos\sqrt{3x+1} \, dx$ $\quad h = \frac{0.4-0}{4} = 0.1$

x	y
0.05	$\cos\sqrt{1.15}$
0.15	$\cos\sqrt{1.45}$
0.25	$\cos\sqrt{1.75}$
0.35	$\cos\sqrt{2.05}$

$\int \approx 0.1 \left\{ \cos\sqrt{1.15} + \cos\sqrt{1.45} + \cos\sqrt{1.75} + \cos\sqrt{2.05} \right\}$

$= 0.122$ (3 s.f.)

6b. $\int_0^1 x\sqrt{3x+1} \, dx$

$u = 3x+1$

$x = \frac{1}{3}(u-1)$

$dx = \frac{1}{3} du$

x	1	0
u	4	1

$\int_1^4 \frac{1}{3}(u-1) u^{1/2} . \frac{1}{3} \, du$

$= \frac{1}{9} \int u^{1/2}(u-1) \, du$

$= \frac{1}{9} \int u^{3/2} - u^{1/2} \, du$

$$= \frac{1}{9}\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_1^4$$

$$= \frac{1}{9}\left(\left(\frac{2}{5}(4)^{5/2} - \frac{2}{3}(4)^{3/2}\right) - \left(\frac{2}{5}(1)^{5/2} - \frac{2}{3}(1)^{3/2}\right)\right)$$

$$= \frac{1}{9}\left(\frac{112}{15} - \left(-\frac{4}{15}\right)\right)$$

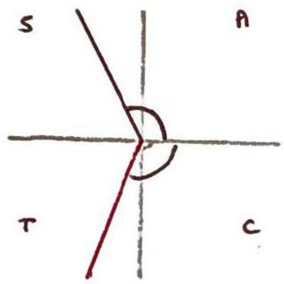
$$= \frac{116}{135}$$

7a.

$\sec x = -5 \qquad 0 < x < 2\pi$

$\cos x = -1/5$

P.V. $x = 1.772^c$



$x = 1.77^c,\ 4.51^c$

7b.

$$\frac{\operatorname{cosec} x}{1+\operatorname{cosec} x} - \frac{\operatorname{cosec} x}{1-\operatorname{cosec} x} = 50$$

$$\frac{\operatorname{cosec} x(1-\operatorname{cosec} x) - \operatorname{cosec} x(1+\operatorname{cosec} x)}{(1+\operatorname{cosec} x)(1-\operatorname{cosec} x)} = 50$$

$$\frac{\operatorname{cosec} x - \operatorname{cosec}^2 x - \operatorname{cosec} x - \operatorname{cosec}^2 x}{1-\operatorname{cosec}^2 x} = 50$$

$$\frac{-2\operatorname{cosec}^2 x}{1-\operatorname{cosec}^2 x} = 50 \qquad (1-\operatorname{cosec}^2 x \equiv -\cot^2 x)$$

$$\frac{-2\operatorname{cosec}^2 x}{-\cot^2 x} = 50$$

$$\frac{\operatorname{cosec}^2 x}{\cot^2 x} = 25$$

$$\operatorname{cosec}^2 x \div \cot^2 x$$

$$\frac{1}{\sin^2 x} \div \frac{\cos^2 x}{\sin^2 x}$$

$$\frac{1}{\sin^2 x} \times \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\therefore \quad \sec^2 x = 25$$

7c.

$$\sec^2 x = 25 \qquad 0 < x < 2\pi$$

$$\sec x = \pm 5$$

$$\sec x = -5$$

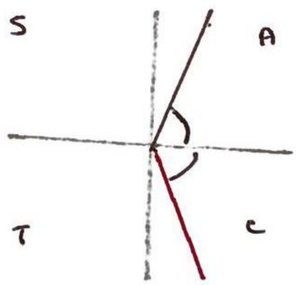
$$x = 1.77^c, \ 4.51^c \ (\text{part a})$$

or

$$\sec x = 5$$

$$\cos x = 1/5$$

P.V. $x = 1.369^c$



$$x = 1.37, \ 4.91^c$$

8a.

$$e^{-2x} = 4$$

$$-2x = \ln 4$$

$$x = -\frac{1}{2}\ln 4$$

8bi.

$$y = 4e^{-2x} - e^{-4x}$$

when $x = 0$, $y = 4e^0 - e^0$

$$= 3 \qquad A\ (0,3)$$

8bii. at B, $y = 0$

$4e^{-2x} - e^{-4x} = 0$. (Since $e^{-2x} > 0$ can divide by it)

$4 - e^{-2x} = 0$

$4 = e^{-2x}$

$x = -\frac{1}{2}\ln 4$ (from 8a)

$= \ln 4^{-1/2}$

$= \ln \frac{1}{2}$

8biii. $y = 4e^{-2x} - e^{-4x}$

$\frac{dy}{dx} = -8e^{-2x} + 4e^{-4x}$

at st. pt. $\frac{dy}{dx} = 0$

$8e^{-2x} = 4e^{-4x}$ $(\times e^{4x})$

$8e^{2x} = 4$

$e^{2x} = \frac{1}{2}$

$2x = \ln(\frac{1}{2})$

$x = \frac{1}{2}\ln(\frac{1}{2})$

8biv. $V = \pi \int_0^{\ln 2} y^2 \, dx$

$y = 4e^{-2x} - e^{-4x}$

$y^2 = (4e^{-2x} - e^{-4x})(4e^{-2x} - e^{-4x})$

$= 16e^{-4x} - 8e^{-6x} + e^{-8x}$

$$V = \pi \int_0^{\ln 2} 16e^{-4x} - 8e^{-6x} + e^{-8x} \, dx$$

$$= \pi \left[-4e^{-4x} + \frac{4}{3}e^{-6x} - \frac{1}{8}e^{-8x} \right]_0^{\ln 2}$$

$$= \pi \left(\left(-4e^{-4\ln 2} + \frac{4}{3}e^{-6\ln 2} - \frac{1}{8}e^{-8\ln 2} \right) - \left(-4e^{0} + \frac{4}{3}e^{0} - \frac{1}{8}e^{0} \right) \right)$$

$$= \pi \left(\left(-\frac{1}{4} + \frac{1}{48} - \frac{1}{2048} \right) - \left(-\frac{67}{24} \right) \right)$$

$$= \pi \left(-\frac{1411}{6144} + \frac{67}{24} \right)$$

$$= \frac{5247}{2048} \pi$$