AQA

A Level

A Level Maths

AQA Core Maths C2 January 2011 Model Solutions

Name:

MME

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Total Marks:

AQA Jan 11 C2

1a. $l = r\theta$

$4 = 5\theta$

$\theta = 0.8^c$

1b. $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}(5)^2(0.8)$

$= 10$

2ai. $8 = 2^p \Rightarrow p = 3$

2aii. $\frac{1}{8} = 2^p$, $\frac{1}{8} = \frac{1}{2^3} \Rightarrow p = -3$

2aiii. $\sqrt{2} = 2^r \Rightarrow r = 1/2$

2b. $\sqrt{2} \times 2^x = \frac{1}{8}$

$2^{1/2} \times 2^x = 2^{-3}$

$2^{x+\frac{1}{2}} = 2^{-3}$

$x + \frac{1}{2} = -3$

$x = -7/2$

3a. $\cos A = \dfrac{c^2 + b^2 - a^2}{2cb}$

$= \dfrac{5^2 + 8^2 - 10^2}{2(5)(8)}$

$\cos A = -\dfrac{11}{80}$

$A = 97.903\ldots$

$= 97.9^\circ$ (1dp)

3bi.

$$\text{Area} = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2}(5)(8)\sin 97.903\ldots$$
$$= 19.8 \quad (3sf)$$

3bii.

Area of triangle $= \frac{1}{2}(b \times h)$ $\quad h = AD$

$$19.8100 = \frac{1}{2}(10h)$$
$$h = \frac{1}{10}(2 \times 19.8100\ldots)$$
$$= 3.96 \quad (3sf)$$

4a.

$$\int_0^{1.5} \sqrt{27x^3+4}\,dx \qquad h = \frac{1.5-0}{3} = 0.5$$

x	y
0	2
0.5	$\sqrt{59/8}$
1	$\sqrt{31}$
1.5	$\sqrt{761/8}$

$$\int \approx \frac{1}{2}(0.5)\left\{\left(2+\sqrt{\frac{761}{8}}\right)+2\left(\sqrt{\frac{59}{8}}+\sqrt{31}\right)\right\}$$
$$= 7.08 \quad (3sf)$$

4b

$f(x) \to f\left(\frac{1}{3}x\right)$ Stretch s.f. 3 in ~~y~~ x direction

$$\sqrt{27x^3+4} \longrightarrow \sqrt{27\left(\frac{1}{3}x\right)^3+4}$$
$$= \sqrt{x^3+4}$$

5a.

$$(1-x)^3 = 1^3 + {}^3C_1\,1^2(-x) + {}^3C_2\,1(-x)^2 + (-x)^3$$
$$= 1 - 3x + 3x^2 - x^3$$

5b.

$$(1+y)^4 = 1 + {}^4C_1\,1^3y + {}^4C_2\,1^2y^2 + {}^4C_3\,1y^3 + {}^4C_4\,y^4$$
$$= 1 + 4y + 6y^2 + 4y^3 + y^4$$

$(1+y)^4 - (1-y)^3$

$= 1 + 4y + 6y^2 + 4y^3 + y^4 - (1 - 3y + 3y^2 - y^3)$

$= 1 + 4y + 6y^2 + 4y^3 + y^4 - 1 + 3y - 3y^2 + y^3$

$= 7y + 3y^2 + 5y^3 + y^4$

$\therefore \quad p = 3 \quad , \quad q = 5$

5c. $\int (1+y)^4 - (1-y)^3 \, dy$

$= \int 7y + 3y^2 + 5y^3 + y^4 \, d\cancel{y}x$

replace y with $x^{1/2}$

$\int 7x^{1/2} + 3x + 5x^{3/2} + x^2 \, dx$

$= \frac{14}{3}x^{3/2} + \frac{3}{2}x^2 + 2x^{5/2} + \frac{1}{3}x^3 + c$

6ai. G.P.

$u_3 = ar^2 = 36$ ①

$u_6 = ar^5 = 972$ ②

'② ÷ ①' $\quad \frac{ar^5}{ar^2} = \frac{972}{36}$

$r^3 = 27$

$r = 3$

6aii. 'sub $r = 3$ into ①'

$a(3)^2 = 36$

$a = 4$

6bi.

$$\sum_{n=1}^{20} u_n = S_{20} = \frac{a(1-r^{20})}{1-r}$$

$$S_{20} = \frac{4(1-3^{20})}{1-3}$$

$$-2S_{20} = 4(1-3^{20})$$

$$S_{20} = -2(1-3^{20})$$

$$S_{20} = 2(3^{20}-1)$$

6bii.

$$u_n = ar^{n-1} = 4 \times 3^{n-1}$$

$$4 \times 3^{n-1} > 4 \times 10^{15} \quad (\div 4)$$

$$3^{n-1} > 10^{15}$$

$$(n-1)\log 3 > 15 \qquad (\log 10^{15} = 15)$$

$$n-1 > \frac{15}{\log 3}$$

$$n > 1 + \frac{15}{\log 3}$$

$$n > 32.43\ldots$$

$$n = 33$$

7a.

$$y = x + 3 + 8x^{-4}$$

$$\frac{dy}{dx} = 1 - 32x^{-5}$$

7b

when $x = 1$, $y = 1 + 3 + 8(1)^{-4}$

$= 12$

$(1, 12)$

$$\frac{dy}{dx} = 1 - 32(1)^{-5}$$

$$= -31$$

$$y - 12 = -31(x-1)$$

$$y + 12 = -31x + 31 \implies y + 31x = 43$$

7c. at minimum $\frac{dy}{dx} = 0$

$1 - \frac{32}{x^5} = 0$

$x^5 = 32$

$x = 2$

when $x = 2$, $y = 2 + 3 + \frac{8}{2^4}$

$= \frac{11}{2}$

$M \; (2, {}^{11}/_{2})$

7di. $\int (x + 3 + 8x^{-4}) \; dx$

$= \frac{1}{2}x^2 + 3x - \frac{8}{3}x^{-3} + c$

7dii. $\left[\frac{1}{2}x^2 + 3x - \frac{8}{3}x^{-3} \right]_1^2$

$= \left(\frac{1}{2}(2)^2 + 3(2) - \frac{8}{3}(2)^{-3} \right) - \left(\frac{1}{2}(1)^2 + 3(1) - \frac{8}{3}(1)^{-3} \right)$

$= \frac{23}{3} - \frac{5}{6}$

$= \frac{41}{6}$

7e. x axis tangent to M

$\therefore$ M moved down 5.5 units

$\Rightarrow$ $k = -5.5$

8a.

$$2\log_k x - \log_k 5 = 1$$

$$\log_k x^2 - \log_k 5 = 1$$

$$\log_k\left(\frac{x^2}{5}\right) = 1$$

$$\frac{x^2}{5} = k$$

8b.

$$\log_a y = \frac{3}{2}$$

$$y = a^{3/2} \quad ①$$

$$\log_4 a = b+2$$

$$a = 4^{b+2} \quad ②$$

'Sub ② into ①'

$$y = \left(4^{b+2}\right)^{3/2}$$

$$= 4^{\frac{3}{2}(b+2)}$$

$$= \left(2^2\right)^{\frac{3}{2}(b+2)}$$

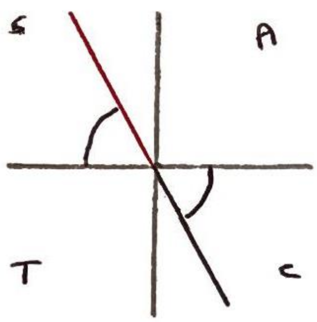
$$= 2^{3(b+2)}$$

$$= 2^{3b+6}$$

9a

$$\tan x = -3 \qquad 0^\circ \leq x \leq 360^\circ$$

P.V. $x = -71.565$



$$x = 288^\circ,\ 108^\circ$$

9bi.

$$7\sin^2\theta + \sin\theta\cos\theta = 6$$

$$7\sin^2\theta + \sin\theta\cos\theta = 6\cos^2\theta + 6\sin^2\theta \qquad \div \cos^2\theta$$

$$7\frac{\sin^2\theta}{\cos^2\theta} + \frac{\sin\theta\cos\theta}{\cos\theta\cos\theta} = \frac{6\cos^2\theta}{\cos^2\theta} + \frac{6\sin^2\theta}{\cos^2\theta}$$

$$= 7\tan^2\theta + \tan\theta = 6 + 6\tan^2\theta$$

$$\tan^2\theta + \tan\theta - 6 = 0$$

9bii.

$$(\tan\theta + 3)(\tan\theta - 2) = 0$$

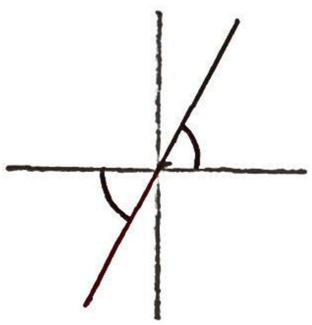
$$\tan\theta = -3$$

$$0 \leq \theta \leq 360°$$

$\theta = 108°, 288°$ (part a))

or $\tan\theta = 2$

P.V. $\theta = 63.435$



$$\theta = 63.4°, 243.4°$$

$\therefore \theta = 63°, 108°, 243°, 288°$ (nearest degree)