AQA

A Level

A Level Maths

AQA Core Maths C1 January 2011 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

AQA Jan 11 C1

1a.

$y = 13 + 18x + 3x^2 - 4x^3$

$\frac{dy}{dx} = 18 + 6x - 12x^2$

1b

at P $x = -1$, $\frac{dy}{dx} = 18 + 6(-1) - 12(-1)^2$

$= 18 - 6 - 12$

$= 0$

$\therefore$ stat. point at P

$12x^2 - 6x - 18 = 0$

$2x^2 - x - 3 = 0$

$(2x - 3)(x + 1) = 0$

$x = -1$ or $x = 3/2$

1ci.

$\frac{d^2y}{dx^2} = 6 - 24x$

when $x = -1$, $\frac{d^2y}{dx^2} = 6 - 24(-1)$

$= 30$

1cii

$30 > 0 \Rightarrow$ minimum at P

2a.

$(3\sqrt{3})^2 = 9(3) = 27$

2b.

$\frac{4\sqrt{3} + 3\sqrt{7}}{3\sqrt{3} + \sqrt{7}} \times (3\sqrt{3} - \sqrt{7})$

$\frac{(4\sqrt{3} + 3\sqrt{7})(3\sqrt{3} - \sqrt{7})}{(3\sqrt{3} + \sqrt{7})(3\sqrt{3} - \sqrt{7})} = \frac{12(3) - 4\sqrt{21} + 9\sqrt{21} - 3(7)}{27 - 7}$

$= \frac{15 + 5\sqrt{21}}{20}$

$= \frac{3 + \sqrt{21}}{4}$

3ai. $3x + 2y = 7$ $\quad$ $C(2, -7)$

$2y = 7 - 3x$

$y = \frac{7}{2} - \frac{3}{2}x \Rightarrow m = -\frac{3}{2}$

3aii. $// \Rightarrow m = -\frac{3}{2}$

$y - -7 = -\frac{3}{2}(x - 2)$

$2y + 14 = -3x + 6$

when it crosses the y axis, $x = 0$

$2y + 14 = -3(0) + 6$

$2y = -8$

$y = -4$

3b. $y = 1 - 4x$ ①

$3x + 2y = 7$ ②

'sub ① into ②'

$3x + 2(1 - 4x) = 7$

$3x + 2 - 8x = 7$

$-5 = 5x$

$x = -1$

'sub $x = -1$ into ①'

$y = 1 - 4(-1)$

$= 5$

$\therefore A(-1, 5)$

3c $E(5, k)$ $\quad$ $C(2, -7)$

$CE = \sqrt{(5-2)^2 + (k - -7)^2}$

$5 = \sqrt{9 + k^2 + 14k + 49}$

$25 = k^2 + 14k + 58$

$k^2 + 14k + 33 = 0 \Rightarrow (k+3)(k-11) = 0$

$k = -3$
or $k = 11$

4ai.

$$y = 14 - x - x^4$$

$$\frac{dy}{dx} = -1 - 4x^3$$

when $x = 1$, $\frac{dy}{dx} = -1 - 4(1)^3$

$$= -1 - 4$$

$$= -5$$

4aii.

m of tangent $= -5$ $\quad P(1,12)$

$$y - 12 = -5(x-1)$$

$$y - 12 = -5x + 5$$

$$y = -5x + 17$$

4bi.

$$\int_{-2}^{1} 14 - x - x^4 \, dx$$

$$= \left[14x - \tfrac{1}{2}x^2 - \tfrac{1}{5}x^5 \right]_{-2}^{1}$$

$$= \left(14(1) - \tfrac{1}{2}(1)^2 - \tfrac{1}{5}(1)^5\right) - \left(14(-2) - \tfrac{1}{2}(-2)^2 - \tfrac{1}{5}(-2)^5\right)$$

$$= \left(14 - \tfrac{1}{2} - \tfrac{1}{5}\right) - \left(-28 - 2 + \tfrac{32}{5}\right)$$

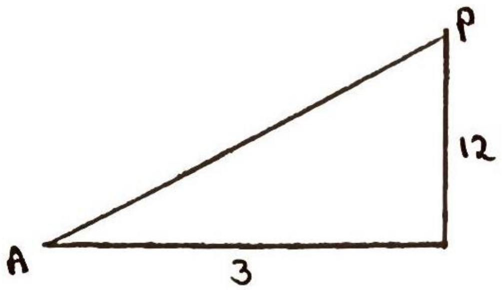
$$= 14 - \tfrac{1}{2} - \tfrac{1}{5} + 28 + 2 - \tfrac{32}{5}$$

$$= 44 - \tfrac{1}{2} - \tfrac{33}{5}$$

$$= \frac{440}{10} - \frac{5}{10} - \frac{66}{10}$$

$$= \frac{369}{10}$$

4bii.



A of Δ $= \frac{1}{2}(3 \times 12)$

$= 18$

Shaded Area = Area under curve - A of Δ

$= \frac{369}{10} - 18$

$= \frac{369}{10} - \frac{180}{10}$

$= \frac{189}{10}$

5ai.

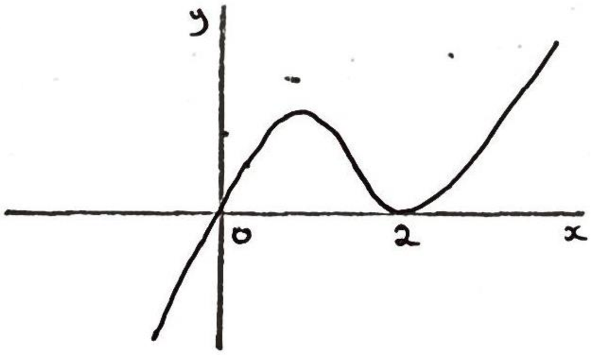
$y = x(x-2)^2$

root at $x = 0$

double root at $x = 2$

when $x = 0$ $y = 0$ $\therefore$ goes through $(0,0)$

$+x^3 \Rightarrow$ shape



5aii.

$x(x-2)^2 = 3$

$x(x^2 - 4x + 4) - 3 = 0$

$x^3 - 4x^2 + 4x - 3 = 0$

5bi.

$$p(x) = x^3 - 4x^2 + 4x - 3$$

$$p(-1) = (-1)^3 - 4(-1)^2 + 4(-1) - 3$$

$$= -1 - 4 - 4 - 3$$

$$= -12 \Rightarrow \text{remainder} = -12$$

5bii.

$$p(3) = (3)^3 - 4(3)^2 + 4(3) - 3$$

$$= 27 - 36 + 12 - 3$$

$$= 0 \Rightarrow (x-3) \text{ is a factor}$$

5biii.

$$\begin{array}{r|l} & x^2 - x + 1 \\ x - 3 & x^3 - 4x^2 + 4x - 3 \\ & \underline{x^3 - 3x^2} \\ & -x^2 + 4x \\ & \underline{-x^2 + 3x} \\ & x - 3 \\ & \underline{x - 3} \\ & 0 \end{array}$$

$$p(x) = (x-3)(x^2 - x + 1)$$

5c.

$$(x-3)(x^2 - x + 1) = 0$$

$x = 3$ or $x^2 - x + 1 = 0$

disc. $(-1)^2 - 4(1)(1)$

$= 1 - 4$

$= -3$

disc. $< 0 \Rightarrow$ no real roots

$\therefore$ only root at $x = 3$

6ai. $C(-3,1)$ $r = \sqrt{13}$

$(x+3)^2 + (y-1)^2 = 13$

6aii.

$x^2 + 6x + 9 + y^2 - 2y + 1 = 13$

$x^2 + y^2 + 6x - 2y - 3 = 0$

6b.

crosses the y axis when $x = 0$

$0^2 + y^2 + 6(0) - 2y - 3 = 0$

$y^2 - 2y - 3 = 0$

$(y+1)(y-3) = 0$

$y = -1$ or $y = 3$

So $A\ (0,-1)$ $B\ (0,3)$

$\therefore$ distance $AB = 4$

6ci

$x = -5$, $y = -2$

$(-5+3)^2 + (-2-1)^2 = 13$

$(-2)^2 + (-3)^2 = 13$

$4 + 9 = 13$ ✓ $\therefore$ $(-5,-2)$ lies on circle

6cii.

$C(-3,1)$ $D(-5,-2)$

$\frac{1--2}{-3--5} = \frac{3}{2}$

6ciii.

m of tangent $= -\cancel{\tfrac{3}{2}}\ \frac{2}{3}$ (since $\perp$ to CD)

$y--2 = -\frac{2}{3}(x--5)$

$3y + 6 = -2x - 10$

$2x + 3y + 16 = 0$

7ai.

$$4 - 10x - x^2 = -(x^2 + 10x - 4)$$
$$= -((x+5)^2 - 25 - 4)$$
$$= 29 - (x+5)^2$$

7aii.

$x = -5$

7bi.

$y = 4 - 10x - x^2 \qquad y = k(4x - 13)$

$$4 - 10x - x^2 = k(4x - 13)$$
$$4 - 10x - x^2 = 4kx - 13k$$
$$x^2 + 4kx + 10x - 13k - 4 = 0$$
$$x^2 + 2x(2k+5) - (13k+4) = 0$$
$$x^2 + 2(2k+5)x - (13k+4) = 0$$

7bii.

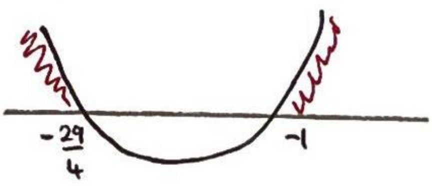
2 distinct points $\Leftrightarrow$ $b^2 - 4ac > 0$

$$(4k+10)^2 - 4(1)(-(13k+4)) > 0$$
$$16k^2 + 80k + 100 + 4(13k+4) > 0$$
$$16k^2 + 80k + 100 + 52k + 16 > 0$$
$$16k^2 + 132k + 116 > 0$$
$$4k^2 + 33k + 29 > 0$$

7biii.

$$(4k+29)(k+1) > 0$$

C.V.s $\quad k = -1 \quad k = -\frac{29}{4}$



$k < -\frac{29}{4}$

$k > -1$