

# GCSE **Mathematics**

43651H Paper 1 Mark scheme

4365 November 2016

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

### **Glossary for Mark Schemes**

Use of brackets

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

М	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
В	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. e.g. accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
[a, b)	Accept values a ≤ value < b
3.14	Accept answers which begin 3.14 e.g. 3.14, 3.142, 3.1416
Q	Marks awarded for quality of written communication

It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

### **Diagrams**

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

### Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

### Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

### Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

### Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

### **Further work**

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

### Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

### Work not replaced

Erased or crossed out work that is still legible should be marked.

### Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

### Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

# Paper 1 Higher Tier

Q	Answer	Mark	Comments	
		1		
	$\pm 2w \text{ or } \pm 18$ or $5w - 3w = 15 + 3$	M1	Terms in <i>w</i> or constant terms	collected
	$2w = 18 \text{ or } -2w = -18 \text{ or } \frac{18}{2}$	A1		
	9	A1ft	ft on $2w = a$ where $a \neq 3$ or 1s or $bw = 18$ where $b \neq 5$ or 3	5
	Ad	ditional G	uidance	
1	2 <i>w</i> = 12 6			M1, A0 A1ft
	8w = 18 2.25 or $\frac{18}{8}$ oe			M1, A0 A1ft
	3w = 12 4			МО
	3 <i>w</i> = 18 6			M1, A0 A0ft
	Embedded answer of 9			M1, A1, A0
	If only decimal answer given must be a	ccurate to	at least 2 dp	

Q	Answer	Mark		Comments	
		I	1		
	1 – (0.2 + 0.3 + 0.15) or 0.65	M1	oe eg	65%	
	0.35	A1	oe eg	35%	
	Ad	ditional G	uidan	се	
	0.2 + 0.3 + 0.15 = 0.2 0.8			Answer follows through	M1 A0
	0.2 + 0.3 + 0.15 = 0.55 1 - 0.55 = 0.25			Method even though answer wrong	M1 A0
2	0.2 + 0.3 + 0.15 = 0.55 0.35		No method seen and answer does not follow through	M0 A0	
	0.65 0.45				M1 A0
	Answer only of 0.65				M1 A0
	0.2 0.8		No addition seen	МО	
	Embedded answer 0.2 + 0.3 + 0.15 + 0.3		M1, A0		
	Embedded answer 0.2 + 0.3 + 0.15 + 0.8 Answer 0.8	8 = 1			M1, A0

3a	[2.3, 2.5]	B1	Ignore x =
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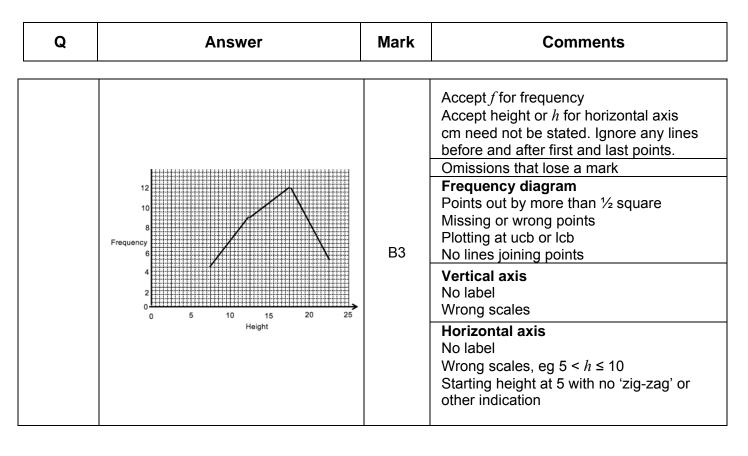
	T				
	Alternative method 1	T	T		
	A triangle drawn on graph or a $y$ and <b>corresponding</b> $x$ length clearly shown ir stated.	M1			
	their $y$ length $\div$ their $x$ length	M1dep	Allow lengths to be $\pm \frac{1}{2}$ small sie $\pm$ 0.2 vertically or $\pm$ 0.1 horizontal		
	5	A1	Only award if $y$ length $\div x$ length ones not round to 5	gth = 5 and	
			Accept $y = 5x - 4$		
	Alternative method 2				
	Substitutes a coordinate value into $y = mx + c$ , eg (2, 6)	M1			
	Shows a correct equation, eg $6 = 2m - 4$	M1dep			
3b	5	A1	Accept $y = 5x - 4$		
	Additional Guidance				
	10.9 ÷ 2.1 = 5			M1 M1dep A0	
	$8 = m \times 2.4 - 4$			M1	
	4 = 2.4 <i>m</i>			M1dep	
	8/3 Additional Guidance of			A0	

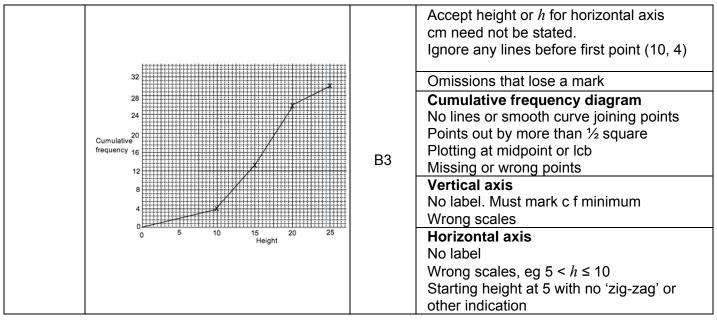
Q	Answer	Mark	Comments
	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		M1 M0dep A0

	6x - 18 - 4x + 20	M1	Three correct terms		
	6x - 18 - 4x + 20	A1	All terms correct		
	2x + 2  or  2(x + 1)	A1ft	ft on M and no further errors  Do not award if incorrect furthe  eg $2x + 2 = 4x$	r work,	
	Additional Guidance				
4	6x - 18 - 4x - 20 $2x - 38$			M1, A0 A1ft	
	5x - 18 - 4x + 20 x + 2			M1, A0 A1ft	
	5x - 18 - 4x - 20 x - 38			M0, A0 A0ft	

Q	Answer	Mark	Comments		
	Frequency polygon or histogram with equal intervals or cumulative frequency diagram.  Vertical axis with equal scales labelled frequency  Horizontal axis clearly numbered and labelled height (units not necessary), may start at 5 with zig-zag on axis.	B1 B1 B1			
	Additional Guidance  The diagram must be an attempt at one of the following. Marks cannot be scored for labels only. If diagram not valid then B0 total.				
5	Frequency 6 10 15 20 25 Height	B3	Accept <i>f</i> for frequency Accept height or <i>h</i> for horizontal axis cm need not be stated  Omissions that lose a mark.  Histogram Heights out by more than ½ square Missing bars Gaps between bars  Vertical axis No label Wrong scales  Horizontal axis No label Wrong scales, eg 5 < <i>h</i> ≤ 10 Starting height at 5 with no 'zig-zag' or other indication		

# Additional Guidance continues on nest page





Q	Answer	Mark	Comments		
	Side of square = 5 Or $5 \times 5 = 25$ oe	B1	May be on diagram		
	400 ÷ 25	M1			
	16	A1	May be on diagram $16 \times 25 = 400$ oe is M1, A1		
	Yes and 5 and their 16	Q1ft	Strand (iii) Conclusion must be length not volume	e based on	
6			ft their 16 if B1, M1 awarded conclusion	and correct	
	Additional Guidance				
	Ignore any volume calculations				
	Square = 5 cm			B1	
	$25 \times 21 = 400$			M1	
	No			A0	
				Q1ft	

Q	Answer	Mark	Comments		
	Alternative method 1				
	BCD = 105	B1			
	DCE = 180 – their 105 or 75	M1	Calculation <b>must</b> be shown of marked on diagram	or correct angle	
	CDE = 180 – (their 75 + 30) or 75	M1dep	Calculation <b>must</b> be shown of marked on diagram	or correct angle	
	DCE = 75 and CDE = 75 and 'two angles equal'	Q1	Strand (ii) Must score B1M2 and have no incorrect angles or calculations seen		
7	Additional Guidance				
	C = 105 C = 180 - 105 = 65 D = 180 - (65 + 30) = 85			B1 M1 M1dep Q0	
	BCD = 75 DCE = 180 - 75 = 105 CDE = 180 - (105 + 30) = 45			B0 M1 M1dep Q0	
	BCD = 105 DCE = 65 CDE = 85 (no method shown)			B1 M0 M0dep Q0	

Alternative methods continued on the next page

Q	Answer	Mark	Comments	
	Alternative method 2			
	ABC = 180 – 105 or 75 or ADC = 180 – 105 = 75	M1	Calculation <b>must</b> be shown or correct angle marked on diagram	
	DCE = their 75	M1dep	their 75 must be the same as their ABC or their ADC	
	CDE = 180 – (their 75 + 30) or 75	M1dep	Calculation <b>must</b> be shown or correct angle marked on diagram	
	DCE = 75 and CDE = 75 and 'two angles equal'	Q1	Strand (ii) Must score M3 and have no incorrect angles or calculations seen	
7 cont	Additional Guidance			
	B = 180 - 105 = 75 C = 105 D = 180 - (105 + 30) = 45		M1 M0dep M0dep Q0	
	ABC (or ADC) = 180 – 105 = 65 DCE = 65 CDE = 85 (no method shown)		M1 M1dep M0dep Q0	
	ABC (or ADC) = 180 - 105 = 75 DCE = 75 CDE = 180 - (75 + 30) = 65		M1 M1dep M1dep Q0	

Alternative methods continued on the next page

Q	Answer	Mark	Comments		
	Alternative method 3				
	BCD = 105	B1			
	CDE = their 105 – 30 or 75	M1	Calculation <b>must</b> be shown or correct angle marked on diagram		
	DCE = 180 – (their 75 + 30) or 75	M1dep	Calculation <b>must</b> be shown or correct angle marked on diagram		
	DCE = 75 and CDE = 75 and 'two angles equal'	Q1	Strand (ii) Must score B1M2 and have no incorrect angles or calculations seen		
7 cont	Additional Guidance				
	C = 105 D = 105 - 30 = 65 C = 180 - (65 + 30) = 85		B1 M1 M1dep Q0		
	BCD = 75 CDE = 75 - 30 = 45 DCE = 180 - (45 + 30) = 105		B0 M1 M1dep Q0		
	BCD = 105 CDE = 65 DCE = 85 (no method shown)		B1 M0 M0dep Q0		

Alternative methods continued on the next page

Q	Answer	Mark	Comments	
	Alternative method 4			
	DCE or CDE = (180 – 30) ÷ 2 or 75	M1	Calculation <b>must</b> be shown of angle marked on diagram	or one correct
	CDE and DCE = their 75	M1dep		
	DCB = 180 – their 75 or 105	M1dep	Calculation <b>must</b> be shown of marked on diagram	or correct angle
7 cont	DCE = 75 and CDE = 75 and DCB = 105 and 'opposite angles of parallelogram equal'	Q1	Strand (ii) Must score M3 and have no incorrect angles or calculations seen	
	Additional Guidance			
	$(180 - 30) \div 2 = 65$ C = 65 and $D = 65C = 115$ (no method shown)			M1 M1dep M0dep Q0
	(180 – 30) ÷ 2 = 75 DCE = 75 and CDE = 75 DCB = 180 – 75 = 105			M1 M1dep M1dep Q0

Q	Answer	Mark	Comments
	2 × (30 + 70) or 200	M1	
	their 200 ÷ 4 or 50	M1 dep	100 ÷ 2 is M2
	their 50 $\times$ their 50 or 2500 or 30 $\times$ 70 or 2100	M1	their 50 must follow M1, M1dep
	400	A1	
	Ac	Iditional G	uidance
	Perimeter = 100 Side of square = 25 2100 – 625 1475		M0 M0dep M1 A0
8	Side of square = $\sqrt{100}$ = 10 2100 - 100 2000		M0 M0dep M1 A0
	30 × 70 = 2400 50 × 50 = 2500 2500 – 2400 = 100		M1 M1dep M1 A0
	Side of square = 25 30 × 70 = 2400 625		M0 M0dep M1 A0
	30 × 70 = 2100		3rd M0
	2100 × 2 = 4200		joid init
			oe
9	6 <i>n</i> + 3 or 3(2 <i>n</i> + 1)	B2	B1 for $6n$ Accept $6 \times n$ or $n \times 6$ but not $n6$

9	6n + 3 or 3(2n + 1)	B2	Accept $6 \times n$ or $n \times 6$ but not $n6$ B1 for $n6 + 3$ Accept any letter
10	(x-10)(x+10) or $(x+10)(x-10)$	B1	
		<u> </u>	

Q	Answer	Mark	Comments
11a	1.6 × 10 <sup>11</sup>	B2	B1 for 2 000 000 × 80 000 B1 for 160 000 000 000 B1 for 16 × 10 <sup>10</sup>
11b	25	B2	B1 2 000 000 ÷ 80 000 or 200 ÷ 8 B1 2.5 × 10 B1 0.25 × 10 <sup>2</sup> B1 for $\frac{2}{8}$ × 10 <sup>2</sup>
		l .	
	Alternative method 1		
	2(2x+3)+(4x-1)	M1	oe one sign or arithmetic error if expanded straight away
	8 <i>x</i> + 5	A1	
	their $(8x + 5) = 20$	M1	oe Not dependent but their 8x + 5 must be linear
	45		oe ft on 1 <sup>st</sup> and 2 <sup>nd</sup> M and one error.
12	1 <u>5</u> 8	A1ft	Decimals must be to at least 2dp, eg 1.875 or 1.88, 1.9 is A0
	Alternative method 2	<u> </u>	
	$2x + 3 + 2x - \frac{1}{2}$	M1	3 terms correct
	$4x + 2\frac{1}{2}$	A1	
	their $(4x + 2\frac{1}{2}) = 10$	M1	oe Not dependent but their $4x + 2\frac{1}{2}$ must be linear
1			

# Alternative method continues on next page

A1ft

15

8

oe ft on 1<sup>st</sup> and 2<sup>nd</sup> M and one error.

Q	Answer	Mark	Comme	nts	
	Alternative method 3				
	$\frac{4x - 1}{2} = 10 - 3 - 2x$	M1			
	4x - 1 = 14 - 4x	M1			
	8 <i>x</i> = 15	A1			
	<u>15</u> 8	A1ft	oe ft on 1 <sup>st</sup> and 2 <sup>nd</sup> M and SC1 Answer from T&I	one error.	
	Ac	ditional g	juidance		
	$2(2x + 3) + 4x - 1$ $4x + 3 + 4x - 1$ $8x + 2 = 20$ $2\frac{1}{4}$		1 error	M1 A0 M1 A1ft	
12 cont	2(2x + 3) + 4x - 1 $4x + 3 + 4x - 1$ $8x + 2 = 20$ $2.75$		2 errors	M1 A0 M1 A0ft	
	$2(2x + 3) + 4x - 1$ $8x + 5$ $8x + 5 = 10$ $\frac{5}{8}$			M1 A1 M0 A0	
	$2(2x + 3) + 4x - 1$ $2x + 6 + 4x - 1$ $6x + 5 = 10$ $\frac{5}{6}$			M1 A0 M0 A0	
	$2x + 3 + 2x - \frac{1}{2}$			M1 A0	
	$4x + 3\frac{1}{2} = 10$		1 error	M1	
	<u>13</u> 8			A1ft	
	Additional Guidance c	ontinues	on next page		

Q	Answer	Mark	Comments
12	2x + 3 + 4x - 1 = 20 $6x + 2 = 20$ $6x = 18$ 3		M0 A0 M1 A0ft
cont	$2x + 3 + 2x - 2 = 10$ $4x + 1 = 10$ $4x = 9$ $2\frac{1}{4}$		M1 A0 M1 A1ft

	Alternative method 1			
	Proportion/percentage fish in lake = $\frac{3}{60}$ or 5%	M1	Could be shown as ratio	
	$400 = 5\%$ or $\frac{5}{100} \times 8000$ or their 5% of 8000	M1dep	oe	
13	100% = 400 × 20 (= 8000) or 400	A1	Calculation of 400 must be clearly shown	
	Alternative method 2			
	Proportion/percentage fish in lake = $\frac{400}{8000}$ or 5%	M1	Could be shown as ratio	
	$\frac{5}{100}$ × 60 or their 5% of 60	M1dep	$\frac{3}{60}$ oe	
	3	A1	Both 5%	

## Alternative method continues on next page

Q	Answer	Mark	Comments	
	Alternative method 3			
	400 ÷ 3 or 8000 ÷ 60	M1		
	8000 ÷ 60 and 400 ÷ 3	M1dep		
	Both 133.333	A1	Must show equivalence clearl	ly
	Alternative method 4			
	8000 ÷ 400 or 60 ÷ 3	M1		
	60 ÷ 3 and 8000 ÷ 40	M1dep		
	Both equal 20	A1	Must show 8000 ÷ 400 = 20 c	clearly
	Ad	ditional G	uidance	
	400 ÷ 3 = 133.3333 8000 ÷ 60 = 800 ÷ 6 = 400 ÷ 3 = 133.333	3		M1 M1dep A1
13 cont	400 ÷ 3 = 133.3333 8000 ÷ 60 = 133.333			M1 M1dep A0
	3 ÷ 60 = 1 ÷ 20 = 0.2 0.2 × 8000 = 400	M1 M1dep A0		
	$3 \div 60 = 0.05$ $0.05 \times 8000 = 400$ She is correct	M1 M1dep A0		
	$3 \div 60 = 0.05$ $0.05 \times 8000 = 0.5 \times 800 = 5 \times 80 = 400$ She is correct			M1 M1dep A1
	60 ÷ 3 = 20 8000 ÷ 400 = 80 ÷ 4 = 20			M1 M1dep A1
	60 ÷ 3 = 20 8000 ÷ 400 = 20			M1 M1dep A0
	3:60 = 1:20 = 2:40 = 4:80 = 400:86	000		M1 M1dep A1
$\frac{400 \times 60}{3} = \frac{824000}{3} = 8000$				M1 M1dep A1

Q	Answer	Mark	Comme	ents
			•	
	(Number of girls =) $\frac{360}{36} \times 5 \text{ or } 50$	M1	oe Check diagram for work	ing
14	Blue eyed girls = $3 \times 5$ or $\frac{108}{360} \times \text{their } 50$ or 15	M1		
	(Number of boys = ) 2 × 4 <sup>2</sup> (× $\pi$ ) or 32 or $\left(\frac{4}{5}\right)^2 \times 50 \ (\times \pi)$	M1	oe	
	Blue eyed boys = their 32 ÷ 4 or 8	M1dep	Dependent on 3 <sup>rd</sup> M	
	23	A1	Must see 32 and 50	
	23, 22, 25 or 24, 21, 25		B2 for 24, 22, 25 B2 for 23, 21, 25 B2 for 23.5, 21.5, 25	
15		B3	B2 for 23, 22, 26 B2 for 24, 21, 26 B2 for 23, 21, 26 B1 for 24, 22, 26 B1 for division by 10 se	en or implied.
	Ad	ditional G	Buidance	
	NB dividing by year will show 215 ÷ 10			В0
16	16		B1 for $64^{\frac{1}{3}} = 4$ B1 for $\sqrt[3]{64 \times 64}$	
		B2	B1 for $\left(64^{\frac{1}{3}}\right)^2$	
			B1 for $(64^2)^{\frac{1}{3}}$ oe	

Comments

	1			
	Alternative method 1			
	$\sqrt{3}^2 + 2 \times \sqrt{3} \times \sqrt{75} + \sqrt{75}^2$	M1	Allow one error	
	3 + 2 × 15 + 75	A1		
	Alternative method 2			
	6√3			
	or $\sqrt{3}^2 \times \left(1 + \sqrt{25}\right)^2$	M1		
17	or $\sqrt{3} + 5\sqrt{3}$			
	$3 \times 6^2$	A1		
	Alternative method 3			
	$\sqrt{108} = 6\sqrt{3}$	M1		
	$\sqrt{3}+5\sqrt{3}=\sqrt{3}+\sqrt{75}$	A1		
	Additional Guidance			
	$(\sqrt{3} + \sqrt{75})(\sqrt{3} + \sqrt{75}) = 9 + \sqrt{225} + \sqrt{3}$	<del>\( \sum_{225} \)</del> + 75 =	= 108	M1, A0
	2(x-1) + x - 3 or $(x-3)(x-1)$	M1		
	2(x-1) + x - 3 = (x - 3)(x - 1)	M1dep		
	$2x-2+x-3 = x^2 - x - 3x + 3$ and $x^2 - 7x + 8 = 0$	A1	oe Do not award if any incorred seen in collecting terms	ct anlegra

**Additional Guidance** 

Mark

Q

18

Ignore any attempts to solve

 $3x - 5 = x^2 - 4x + 3$ 

 $x^2 - 7x + 8 = 0$ 

2(x-1) + x - 3 = (x - 3)(x - 1)

Answer

M1, M1dep

Α1

Q	Answer	Mark	Comments	
	(2x+5)(3x+1) = 8	M1		
	$6x^2 + 17x - 3 = 0$	A1		
	(6x - 1)(x + 3) or $(ax + c)(bx + d)$ where $ab = 6$ and $cd = 3$	M1	An attempt to solve their quadration $6x^2 + 17x + 5 = 0$ .  Must take as far as a correct fact or correct substitution into formul	orisation
	or $\frac{-17 \pm \sqrt{361}}{12}$ or $\frac{17 \pm \sqrt{361}}{12}$			u.
	or $\frac{-17 \pm \sqrt{217}}{12}$			
19	1	A1ft	If negative value (–3 if correct) gi award A1	ven do not
	$\frac{1}{6}$	AIII	ft their solution if only positive value given and evaluated to 2dp at least	
	Additional Guidance			
	(2x + 5)(3x + 1) = 8		M1	
	$6x^2 + 8x + 2x + 5 = 8$ $6x^2 + 10x - 3 = 0$		A0	
	$\begin{array}{c c} -10 \pm \sqrt{172} \\ \end{array}$		M1	
	12		A0	
	(2x + 5)(3x + 1) = 8		M1	
	$6x^{2} + 5x + 2x + 5 = 8$ $6x^{2} + 7x - 3 = 0$		A0	
	(2x+3)(3x-1)=0		M1	
	$\frac{1}{3}$		A1ft	
20a	130	B1		

В1

**20**b

95

Q	Answer	Mark	Comments
20c	BAD = 50 or BOD (reflex) = 260 or ADC = 75 BOD = 100 OBC = 65 or ODC = 65	B1 B1 B1 Additional G	
		Additional G	uidance
	$\frac{150}{360} \times 2 \times \pi \times 6$ or $5\pi$ or [15.5, 15.71]	M1	oe
21	$2  imes$ their $5\pi$ or $\frac{300}{360}  imes 2  imes \pi  imes 6$	M1dep	oe NB $\frac{300}{360}  imes 2  imes \pi  imes 6$ is M2
	10π or [31, 31.42]	A1	
	their 10π + 18 or [49, 49.42]	A1ft	SC1 18 or 6 + 6 + 3 + 3 seen