AQA, Edexcel, OCR

A Level

A Level Mathematics

Parametric Equations (Answers)

Name:

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Total Marks:

C4- Parametric Equations – Answers

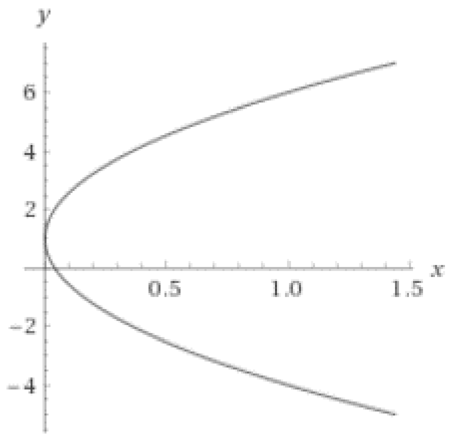
AQA, Edexcel, OCR

1) **Sketch the parametric curve for the following set of equations**

$$\boldsymbol{x = t^2, \quad y = 5t + 1, \quad -2 \leq t \leq 2}$$

[1 mark for x/y graph axes]

[1 mark for correct shape]



2) **Eliminate the parameter from the following set**

$$\boldsymbol{x = 2t^2 + 4} \qquad \textbf{(1)}$$

$$\boldsymbol{y = t + 1} \qquad \textbf{(2)}$$

[1 mark]

Rearranging (2) gives **(3)**

$$t = y - 1$$

Substituting (3) into (1)

$$x = 2(y-1)^2 + 4$$

$$= 2(y^2 - 2y + 2) + 4$$

3) A curve, C, has the parametric equations

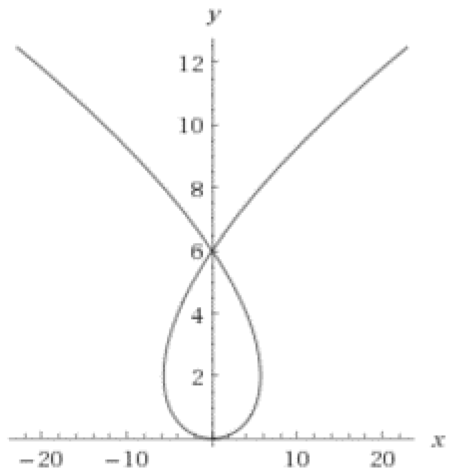
$$x = t^3 - 6t, \quad y = t^2$$

where t is a parameter.

[1 mark for x/y graph axes]

[1 mark for correct shape]

i) Plot y against x for $-2 \leq t \leq 2, t \in \mathbb{Z}$



Point P has the value $t = 1$

ii) Find the coordinates of P. (1)

(2)

$$x = t^3 - 6t$$

$$y = t^2$$

[1 mark]

Substituting $t = 1$ into (1) and (2) gives

$$x = 1^3 - 6(1)$$

$$x = -5$$

$$y = 1^2$$

$$y = 1$$

$$\therefore P = (-5, 1)$$

A square, S, has an edge S_1 that is tangent to C at point P.

iii) Show that the equation for this edge is $3y + 2x + 7 = 0$ **(3)**

[1 mark]

Here, we are defining a line that passes through (-5,1).

[1 mark]

The gradient is given by the chain rule in the form

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx}$$

Using (2) we can define

$$\frac{dy}{dx} = 2t$$

Using (1) we can define

$$\frac{dx}{dt} = 3t^2 - 6$$

$$\therefore \frac{dt}{dx} = \frac{1}{3t^2 - 6}$$

$$\therefore \frac{dy}{dx} = \frac{2t}{3t^2 - 6}$$

[1 mark]

At this point $t = 1$ so we can define the gradient of the S_1 as

$$m = \frac{2(1)}{3(1^2) - 6} = -\frac{2}{3}$$

[1 mark]

Substituting this into $y - y_1 = m(x - x_1)$, with $P = (-5,1)$ gives

$$y - 1 = -\frac{2}{3}(x - -5)$$

$$3y - 3 = -2x - 10$$

$$3y + 2x + 7 = 0$$

(4)

The same edge intersects the curve at second point, Q.

iv) What are the coordinates of this point?

[1 mark]

Intersection of two lines involves solving simultaneous equations.

Substitute (1) and (2) into (3) to give (3) in parametric form.

$$3y + 2x + 7 = 0$$

$$3(t^2) + 2(t^3 - 6t) + 7 = 0$$

$$3t^2 + 2t^3 - 12t + 7 = 0$$

$$2t^3 + 3t^2 - 12t + 7 = 0$$

[1 mark]

Using the factor theorem, we know that $t - 1$ is factor of (4).

Using polynomial long division, we get

$$(t-1)(2t^2 + 5t - 7) = 0$$

$$(t-1)(2t+7)(t-1) = 0$$

This gives us potential solutions at $t = -\frac{7}{2}$ or 1. We know that 1 is the solution previously found (given to us) to that $t = -\frac{7}{2}$ is where Q is.

[1 mark]

Substituting this into (1) and (2)

$$x = \left(-\frac{7}{2}\right)^3 - 6\left(-\frac{7}{2}\right) = -\frac{343}{8} + \frac{42}{2} = -\frac{175}{8}$$

$$y = -\frac{7}{2}^2 = \frac{49}{4}$$

$$\therefore Q = (-\frac{175}{8}, \frac{49}{4})$$

4) **The following parametric equations**

$$\boldsymbol{x = t^2 - 4t} \quad (1)$$

$$\boldsymbol{y = t^3 - 4t} \quad (2)$$

define a curve, $\boldsymbol{C}$, that cross the x-axis thrice.

i) **One of the points, $\boldsymbol{N}$, at which it crosses is (0,0). Find the one where $\boldsymbol{x > 0}$.**

[1 mark]

At the x-axis, $y = 0$. Subbing in y=0 into (2) gives

$$0 = t(t^2 - 4t)$$

$$t^2 = 4$$

$$t = \pm 2$$

Substituting $t = 2$ into (1) gives

$$x = 2^2 - 4(2) = 4 - 8 = -4$$

which is not $x > O$

[1 mark]

Substituting $t = -2$ into (1) gives

$$x = (-2)^2 - 4(-2) = 4 + 8 = 12$$

$$N = (12,0)$$

From the *origin* to the *N*, a region, *R*, of the of the plane is enclosed by C and the x-axis.

ii) **Find the area of *R*.**

[1 mark]

The area is given by the integral of the curve.

$$R = \int_{0}^{12} t^3 - 4t \ dx$$

[1 mark]

However, we need the integral with respect to dt.

$$R = \int_{0}^{12} t^3 - 4t \frac{dx}{dt} dt$$

where, differentiating (1) gives us

$$\frac{dx}{dt} = 2t - 4$$

[1 mark]

We also need to change the limits of the integral, we know that when

$$x = 0, t = 0$$

$$x = 12, t = -2$$

$$\therefore R = \int_{0}^{-2} t^3 - 4t\,(2t - 4) dt$$

$$= \int_{0}^{-2} 2t^4 - 8t^2 + 4t^3 - 16t \ dt$$

[1 mark]

$$= \left[\frac{16t^5}{5} + t^4 - \frac{8t^3}{3} - 8t^2\right]_0^{-2}$$

$$= \left(\frac{16(-2)^5}{5} + (-2)^4 - \frac{8(-2)^3}{3} - 8(-2)^2\right) - \left(\frac{16(0)^5}{5} + 0^4 - \frac{8(0)^3}{3} - 8(0)^2\right)$$

$$= -97 \ square \ units$$

$$= 97 \ units^2$$

5) i) **State the parameterisation of the of the circle $x^2 + y^2 = 9$**

Recall that $\sin^2 x + \cos^2 x = 1,$ therefore we can write

$$x = \text{asin}(t)$$

$$y = \text{acos}(t)$$

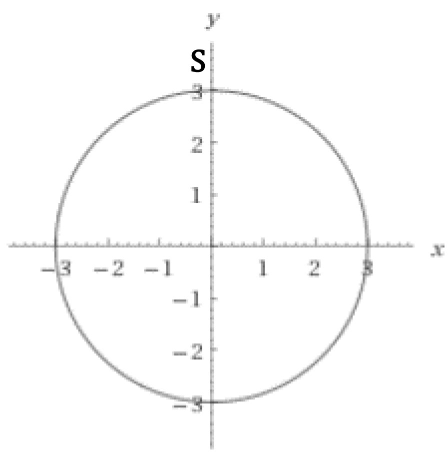
[1 mark for each correctly written- 2 max]

For this example

$$x = 3\sin(t)$$

$$y = 3\cos(t)$$

iii) **Plot this circle and indicate the starting point of motion, S, and the direction of motion as clockwise or anticlockwise.**



[1 mark]

Motion starts at $t = 0$

$$x = 3\sin(0) = 0$$

$$y = 3\cos(0) = 1$$

$$\therefore S = (0,1)$$

[1 mark]

Any increase in t will make it positive and the values of $a\sin(t)$ and $bcos(t)$ will also be positive meaning, the motion will be modelling in Quadrant 1, positive, making it clockwise.