AQA, Edexcel, OCR, MEI

A Level

A Level Mathematics

C4 Vectors

Name:

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Total Marks: /107

C4 - Vectors
MEI, OCR, AQA, Edexcel

1. State whether each of the following quantities are scalars or vectors. For parts g) and h), you are given that $\boldsymbol{a}$ and $\boldsymbol{b}$ are vectors:

   (a) $\boldsymbol{i} + 2\boldsymbol{j}$. [1]

   (b) 2. [1]

   (c) $3\boldsymbol{i}$. [1]

   (d) $(1, 2)$. [1]

   (e) $\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$. [1]

   (f) $|3\boldsymbol{i} + 2\boldsymbol{j}|$. [1]

   (g) $\boldsymbol{a} \cdot \boldsymbol{b}$. [1]

   (h) $\boldsymbol{a} - \boldsymbol{b}$. [1]

2. Simplify the following expressions:

   (a) $\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$. [1]

   (b) $2\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$. [1]

   (c) $3\boldsymbol{i} + 2\boldsymbol{j} + 5\boldsymbol{i} + 8\boldsymbol{j}$. [1]

   (d) $6(2\boldsymbol{i} + \boldsymbol{j})$. [1]

3. Evaluate the following expressions:

   (a) $|\boldsymbol{i} + \boldsymbol{j}|$. [2]

   (b) $|3\boldsymbol{i} + 4\boldsymbol{j}|$. [2]

   (c) $|2\boldsymbol{i} + 8\boldsymbol{j}|^2$. [2]

   (d) $\left|\cos t\boldsymbol{i} + \sin t\boldsymbol{j} + \sqrt{3}\boldsymbol{k}\right|$. [3]

4. Calculate each of the scalar (dot) products below. Also specify whether the vectors are orthogonal to one another or not:

   (a) $\begin{pmatrix}1\\1\\1\end{pmatrix} \cdot \begin{pmatrix}1\\1\\1\end{pmatrix}$ [2]

   (b) $\begin{pmatrix}1\\2\\1\end{pmatrix} \cdot \begin{pmatrix}-1\\1\\-1\end{pmatrix}$ [2]

   (c) $\begin{pmatrix}3\\4\\5\end{pmatrix} \cdot \begin{pmatrix}1\\-8\\2\end{pmatrix}$ [2]

   (d) $\begin{pmatrix}-2\\5\\3\end{pmatrix} \cdot \begin{pmatrix}13\\4\\2\end{pmatrix}$ [2]

   (e) $(3\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}) \cdot (\boldsymbol{i} + \boldsymbol{j} + \sqrt{3}\boldsymbol{k})$ [2]

5. Find the distance between the following lines and the location of the mid point between them:

   (a) $\begin{pmatrix}2\\1\\3\end{pmatrix}$ and $\begin{pmatrix}1\\1\\1\end{pmatrix}$ [3]

   (b) $\begin{pmatrix}4\\1\\6\end{pmatrix}$ and $\begin{pmatrix}3\\2\\1\end{pmatrix}$ [3]

   (c) $\begin{pmatrix}-1\\-4\\0\end{pmatrix}$ and $\begin{pmatrix}8\\-2\\1\end{pmatrix}$ [3]

6. Consider the position vectors $\overrightarrow{OA} = \begin{pmatrix}1\\2\\1\end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix}2\\6\\1\end{pmatrix}$:

   (a) Calculate $\left|\overrightarrow{AB}\right|$. [3]

   (b) The point C lies on the line between A and B and divides it in the ratio $2 : 3$. Work out the coordinates of C. [3]

7. Write the equation of the line through $\begin{pmatrix}1\\1\\1\end{pmatrix}$ in the direction $\begin{pmatrix}1\\2\\1\end{pmatrix}$ in:

   (a) Vector form. [1]

   (b) Cartesian form. [3]

8. Consider two points, $A = \begin{pmatrix}1\\2\\2\end{pmatrix}$ and $B = \begin{pmatrix}2\\0\\4\end{pmatrix}$:

   (a) Write the equation of the line through A and B in vector form. [2]

   (b) Write the equation of the line through A and B in cartesian form. [3]

9. Write the following equations of lines in vector form:

   (a) $y = 2x$. [2]

   (b) $y = 3x + 1$. [2]

   (c) $\frac{x-1}{2} = \frac{y-2}{3} = z$. [3]

10. Find the equation of the following planes in *vector form*:

    (a) The plane through $\begin{pmatrix}1\\1\\1\end{pmatrix}$ with direction vectors $\begin{pmatrix}1\\2\\1\end{pmatrix}$ and $\begin{pmatrix}3\\0\\0\end{pmatrix}$. [1]

    (b) The plane containing the origin and the points $\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\begin{pmatrix}4\\2\\1\end{pmatrix}$ and $\begin{pmatrix}3\\1\\0\end{pmatrix}$. [2]

11. Find the equation of the following planes in *cartesian form*:

    (a) The plane through $\begin{pmatrix}1\\1\\1\end{pmatrix}$ with normal vector $\begin{pmatrix}1\\0\\0\end{pmatrix}$. [3]

    (b) The plane through $\begin{pmatrix}1\\2\\1\end{pmatrix}$ with normal vector $\begin{pmatrix}\frac{3}{5}\\0\\\frac{2}{5}\end{pmatrix}$. [3]

12. *Challange:* Find the cartesian equation of the plan containing the points $\begin{pmatrix}5\\4\\3\end{pmatrix}$, $\begin{pmatrix}4\\2\\3\end{pmatrix}$ and $\begin{pmatrix}9\\2\\0\end{pmatrix}$. [6]

13. Find the points of intersection between the following lines/planes:

    (a) The line $\boldsymbol{r_1} = s_1\begin{pmatrix}1\\2\end{pmatrix}$ and the line $\boldsymbol{r_2} = s_2\begin{pmatrix}1\\3\end{pmatrix} - \begin{pmatrix}\frac{1}{3}\\0\end{pmatrix}$. [3]

    (b) The plane $3x + y = 25$ and the line $\boldsymbol{r} = \begin{pmatrix}2\\1\end{pmatrix} s$. [4]

14. Calculate the angle between the following vectors. Give you answers in *degrees* to two decimal places where necessary:

    (a) $3\boldsymbol{i} + 2\boldsymbol{j}$ and $3\boldsymbol{i}$. [2]

    (b) $2\boldsymbol{i} + 7\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}$. [2]

    (c) $\begin{pmatrix}1\\1\\0\end{pmatrix}$ and $\begin{pmatrix}2\\3\\1\end{pmatrix}$. [2]

    (d) $\begin{pmatrix}3\\1\\2\end{pmatrix}$ and $\begin{pmatrix}-2\\2\\-2\end{pmatrix}$. [2]

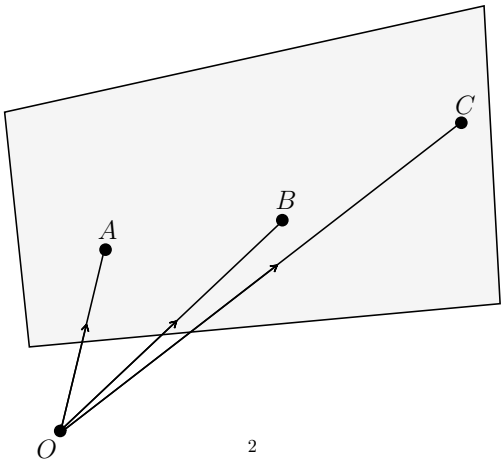
15. Calculate the angle between the following planes. Give you answers in *degrees* to two decimal places where necessary:

    (a) $2x + y + z = 4$ and $3x - 2y - z = 16$. [2]

    (b) $3x + y + 2z = 8$ and $-2x + 2y + 2z = 4$. [2]

Turn over

16. You are given that the points A, B, C all lie in a plane, where $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix}$.



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(a) Calculate $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [2]

(b) Hence, or otherwise calculate the *unit* normal vector to the plane containing A, B and C. [3]

(c) Find the equation of the plane in *cartesian* form. [3]

(d) Does the point $\begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix}$ lie on the plane? [1]

(e) Does the point $\begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$ lie on the plane? [1]

(f) Does the point $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ lie on the plane? [1]