AQA, Edexcel, OCR, MEI

A Level

A Level Mathematics

C4 Calculus

Name:

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Total Marks: /75

C4 - Calculus
MEI, OCR, AQA, Edexcel

1. Evaluate the following integrals by expressing the integrand in partial fractions. *Remember to include a constant of integration*:

   (a) $\int \frac{1}{(x+1)(x+2)}\,dx$. [3]

   (b) $\int \frac{x}{(x+1)(x+3)}\,dx$. [3]

   (c) $\int \frac{x^2}{(x+1)(x+2)}\,dx$. [3]

   (d) $\int \frac{1}{(x+1)(x+2)(x+3)}\,dx$. [3]

   (e) $\int \frac{x^3}{(x-4)(x+2)}\,dx$. [4]

   (f) $\int \frac{2x}{(x-1)^2(x+4)}\,dx$. [4]

2. Find the volumes of the solids generated by revolving the following functions around the x axis:

   (a) $y = x^2, \qquad 0 \leq x \leq 2$. [2]

   (b) $y = x^2 + 2x + 1, \qquad 0 \leq x \leq 2$. [2]

   (c) $y = \sqrt{\sin x \cos x}, \qquad 0 \leq x \leq \frac{\pi}{2}$. [3]

   (d) $y = e^{2x}, \qquad 0 \leq x \leq 1$. [2]

   (e) $y = \frac{3}{1-x}, \qquad -2 \leq x \leq 0$. [3]

3. Find the volumes of the solids generated by revolving the following functions around the y axis:

   (a) $y = \frac{x}{2}, \qquad 0 \leq y \leq 2$. [2]

   (b) $y = \sin^{-1} x, \qquad 0 \leq y \leq \frac{\pi}{2}$. [2]

   (c) $y = \frac{1}{x}, \qquad 1 \leq y \leq 2$. [2]

   (d) $y = \frac{1}{2}\ln x, \qquad 0 \leq y \leq 1$. [3]

4. The gradient function of a function $y(x)$ is given by $2x$:

   (a) Write the information above as a first order differential equation. [2]

   (b) Find the general solution of the ODE in your answer to part a). [2]

   (c) Give the solution of the ODE that satisfies the condition $y(1) = 3$. [1]

5. Consider the function $y = \sin x + x$:

   (a) Write down an ODE that satisfies the above equation. [2]

   (b) Find the general solution of the ODE by integrating your answer to part a). [2]

6. You are given that $\frac{dx}{dt} = \frac{1}{2\sqrt{t}}$ and $\frac{dy}{dt} = 2t$, for some parametric equations $x(t)$ and $y(t)$:

   (a) Using the fact that $x(t) = \sqrt{t}$, find an ODE involving x and y only. *Hint:* $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{1}{\frac{dx}{dt}}$. [4]

   (b) Find the general solution of the ODE in your answer to part a). [2]

7. Solve the following differential equations:

   (a) $\frac{dy}{dx} = 4x$. [2]

   (b) $\frac{dy}{dx} = \cos x$. [2]

   (c) $\frac{dy}{dx} = 2xe^{x^2}$. [2]

   (d) $\frac{dy}{dx} = x^2 y$. [3]

   (e) $\frac{dy}{dx} = \frac{y}{x} + y$. [3]

8. Consider the ODE $\frac{dy}{dx} = -\frac{x}{y}$:

   (a) Find the general solution to this ODE. [4]

   (b) Give the exact solution satisfying $y(2) = 0$. [2]

   (c) What does your solution to part b) give the equation of geometrically? [1]