

AQA, Edexcel, OCR, MEI

A Level

A Level Mathematics

C4 Calculus (Answers)

Name:

M M E

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Total Marks: /75

1. Evaluate the following integrals by expressing the integrand in partial fractions. *Remember to include a constant of integration:*

$$(a) \int \frac{1}{(x+1)(x+2)} dx = \ln(x+1) - \ln(x+2) + c. \quad [3]$$

$$(b) \int \frac{x}{(x+1)(x+3)} dx = \frac{3}{2} \ln(x+3) - \frac{1}{2} \ln(x+1) + c. \quad [3]$$

$$(c) \int \frac{x^2}{(x+1)(x+2)} dx = x + \ln(x+1) - 4 \ln(x+2) + c. \quad [3]$$

$$(d) \int \frac{1}{(x+1)(x+2)(x+3)} dx = \frac{1}{2} \ln(x+1) - \ln(x+2) + \frac{1}{2} \ln(x+3) + c. \quad [3]$$

$$(e) \int \frac{x^3}{(x-4)(x+2)} dx = \frac{1}{6} [3(x^2 + 4x - 32) + 64 \ln(x-4) + 8 \ln(x+2)] + c. \quad [4]$$

$$(f) \int \frac{2x}{(x-1)^2(x+4)} dx = \frac{2}{25} \left[\frac{5}{1-x} + 4 \ln(1-x) - 4 \ln(x+4) \right] + c. \quad [4]$$

2. Find the volumes of the solids generated by revolving the following functions around the x axis:

$$(a) \frac{32\pi}{5}. \quad [2]$$

$$(b) \frac{242\pi}{5}. \quad [2]$$

$$(c) \frac{\pi}{2}. \quad [3]$$

$$(d) \frac{\pi}{4}(e^4 - 1). \quad [2]$$

$$(e) 2\pi. \quad [3]$$

3. Find the volumes of the solids generated by revolving the following functions around the y axis:

$$(a) \frac{32\pi}{3}. \quad [2]$$

$$(b) \frac{\pi^2}{4}. \quad [2]$$

$$(c) \frac{\pi}{2}. \quad [2]$$

$$(d) \frac{\pi}{4}(e^4 - 1). \quad [3]$$

4. The gradient function of a function $y(x)$ is given by $2x$:

(a) $\frac{dy}{dx} = 2x$. [2]

(b) $y = x^2 + c$. [2]

(c) $y = x^2 + 2$. [1]

5. Consider the function $y = \sin x + x$:

(a) $\frac{dy}{dx} = \cos x + 1$. [2]

(b) $y = \sin x + x + c$. [2]

6. You are given that $\frac{dx}{dt} = \frac{1}{2\sqrt{t}}$ and $\frac{dy}{dt} = 2t$, for some parametric equations $x(t)$ and $y(t)$:

(a) $\frac{dy}{dx} = 4x^3$. [4]

(b) $y = x^4 + c$. [2]

7. Solve the following differential equations:

(a) $y = 2x^2 + c$. [2]

(b) $y = \sin x + c$. [2]

(c) $y = e^{x^2} + c$. [2]

(d) $y = Ae^{\frac{x^3}{3}}$. [3]

(e) $y = Axe^x$. [3]

8. Consider the ODE $\frac{dy}{dx} = -\frac{x}{y}$:

(a) $x^2 + y^2 = c$. [4]

(b) $x^2 + y^2 = 4$ [2]

(c) This is the equation of a circle centred at the origin of radius 2. [1]